CONTACT OF RACK AND PINION TOOTH FLANKS

Martin Kratochvil

University of West Bohemia

Abstract

The paper is devoted to creation of tooth geometry of rack and pinion mechanism using MATLAB. The aim is to create geometry of pinion tooth based on known geometry of rack tooth. Geometry of rack tooth flank surface is described by position vector and matrix transformation. Mutual movement of rack and pinion is determined by matrix transformations. Then, normal vector of rack tooth flank surface and tangent vector of movement are created. Calculation of contact position is done using numeric equation solver. The solution is contact angle, which determines position of contact. Geometry of pinion tooth can be calculated using position vector of rack tooth and contact angle.

1 Rack tooth flank surface

The geometry of the rack tooth flank surface is defined in the standard CSN 01 4607 [1]. For the purposes of calculation of the pinion gear tooth flank surface it is necessary to create the position vector of the rack tooth flank. The diameter of the pinion and the tooth size are determined by the number of the pinion teeth z and modulus m.

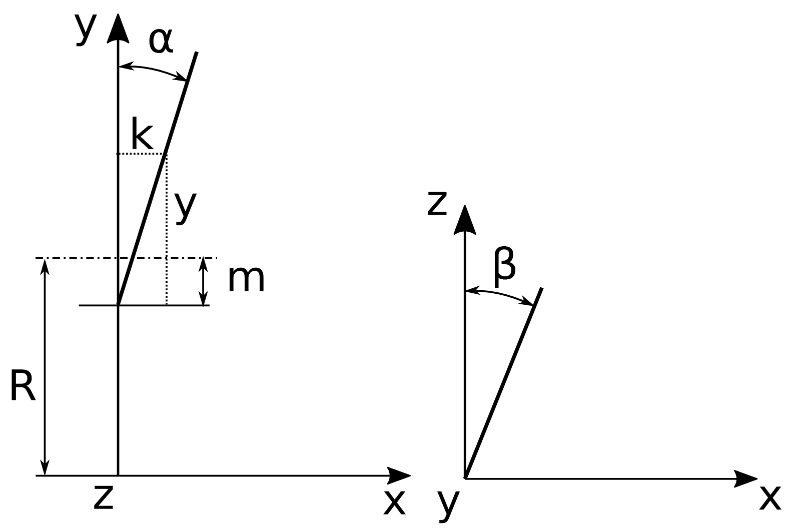

Figure 1: Rack tooth profile

Equation of *tooth profile* in XY plane

$$\tan\alpha = \frac{k}{y} \tag{1}$$

Equation of *slope* in XZ plane

$$\tan\beta = \frac{x}{z} \tag{2}$$

For reason of clarity of matrices, substitution is used:

$$A = \frac{1}{\tan\alpha} \tag{3}$$

$$B = \tan\beta \tag{4}$$

$$R = \frac{mz}{2} \tag{5}$$

The position vector is created using equations (1), (2) and substitution by (3), (4), (5)

$$\mathbf{s}_H = \begin{bmatrix} k + zB \\ kA + R \\ z \end{bmatrix} \tag{6}$$

where k is curve parameter and z is position at Z axis. The first value of the curve parameter k should be:

$$k_0 = m \tan \alpha \tag{7}$$

The next step is to determine the normal vector of the rack tooth flank surface. It is based on two tangent vectors, the tangent vector of the profile curve (8) and the tangent vector of slope (9).

$$\mathbf{t}_1 = \frac{d\mathbf{s}_H}{d\rho} = \begin{bmatrix} 1 \\ A \\ 0 \end{bmatrix} \tag{8}$$

$$\mathbf{t}_2 = \begin{bmatrix} \sin\beta \\ 0 \\ \cos\beta \end{bmatrix} \tag{9}$$

The normal vector of the rack tooth surface is cross product of the two tangent vectors:

$$\mathbf{n}_H = \mathbf{t}_1 \times \mathbf{t}_2 = \begin{bmatrix} A \\ -1 \\ 0 \end{bmatrix} \times \begin{bmatrix} \sin\beta \\ 0 \\ \cos\beta \end{bmatrix} = \begin{bmatrix} A\cos\beta \\ -\cos\beta \\ -A\sin\beta \end{bmatrix} \tag{10}$$

2 Transformation matrices

The transformation matrices are used to translate or rotate a vector. For the purpose of calculation of the mating profile of the pinion gear, the transformation matrices describes the motion of the rack.

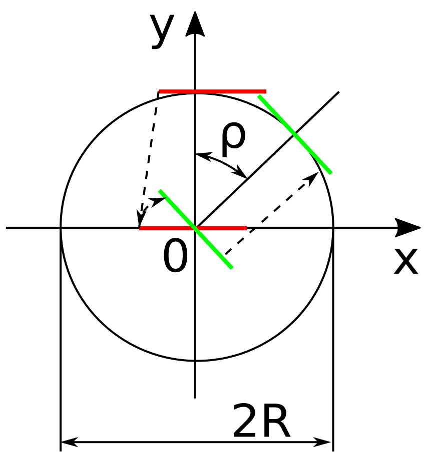

Figure 2: Vector transformations

The motion of the rack is described by three matrix transformations. The first matrix describes the translation to origin and the relative displacement during rolling:

$$\mathbf{T}_{pos} = \begin{bmatrix} 1 & 0 & 0 & -R\rho \\ 0 & 1 & 0 & -R \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{11}$$

The second matrix describes rotation clockwise by angle ρ:

$$\mathbf{T}_{rot} = \begin{bmatrix} \cos\rho & \sin\rho & 0 & 0 \\ -\sin\rho & \cos\rho & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{12}$$

The last transformation is translation to involute rolling point:

$$\mathbf{T}_{pol} = \begin{bmatrix} 1 & 0 & 0 & R\sin\rho \\ 0 & 1 & 0 & R\cos\rho \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{13}$$

The transformation that describes the motion of the rack is the dot product of these three matrices:

$$\mathbf{T}_{celk} = \mathbf{T}_{pos} \cdot \mathbf{T}_{rot} \cdot \mathbf{T}_{pol} = \begin{bmatrix} \cos\rho & \sin\rho & 0 & -R\rho\cos\rho \\ -\sin\rho & \cos\rho & 0 & R\rho\sin\rho \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{14}$$

3 Vector transformations

The rack rolling on the pinion is described by the dot product of matrix (14) and vector (6):

$$\mathbf{s}_{Hb} = \mathbf{T}_{celk} \cdot \mathbf{s}_H = \begin{bmatrix} \cos\rho & \sin\rho & 0 & -R\rho\cos\rho \\ -\sin\rho & \cos\rho & 0 & R\rho\sin\rho \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} k + zB \\ kA + R \\ z \\ 1 \end{bmatrix} \tag{15}$$

$$\mathbf{s}_{Hb} = \begin{bmatrix} (k + zB)\cos\rho + (kA + R)\sin\rho - R\rho\cos\rho \\ -(k + zB)\sin\rho + (kA + R)\cos\rho + R\rho\sin\rho \\ z \\ 1 \end{bmatrix} \tag{16}$$

The direction of the normal vector (10) of the rack tooth flank varies with the rolling of the rack. Because it is a directional vector, to preserve direction, only the matrix of rotation (12) can be used:

$$\mathbf{n}_{Hb} = \mathbf{T}_{rot} \cdot \mathbf{n}_H = \begin{bmatrix} \cos\rho & \sin\rho & 0 & 0 \\ -\sin\rho & \cos\rho & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} A\cos\beta \\ -\cos\beta \\ -A\sin\beta \\ 1 \end{bmatrix} \tag{17}$$

$$\mathbf{n}_{Hb} = \begin{bmatrix} (A\cos\rho - \sin\rho)\cos\beta \\ (-A\sin\rho - \cos\rho)\cos\beta \\ -A\sin\beta \\ 1 \end{bmatrix} \tag{18}$$

4 Tangent vector of trajectory

The transformed position vector of the rack tooth flank surface $\mathbf{s}_{Hb}$ (16) describes the trajectory of every point of the rack tooth flank surface. To calculate the contact angle, it is necessary to calculate the tangent vector of trajectory:

$$\mathbf{t}_{sHb} = \frac{d\mathbf{s}_{Hb}}{d\rho} = \begin{bmatrix} -(k + zB)\sin\rho + (kA + R)\cos\rho + R(\rho\sin\rho - \cos\rho) \\ -(k + zB)\cos\rho - (kA + R)\sin\rho + R(\rho\cos\rho + \sin\rho) \\ 0 \\ 1 \end{bmatrix} \tag{19}$$

5 Calculation of contact angle

The point of the rack tooth flank surface is in contact, when its normal vector $\mathbf{n}_{Hb}$ and tangent vector $\mathbf{t}_{sHb}$ of trajectory are perpendicular. The condition of vector perpendicularity is the zero dot product of these vectors. The equation is a nonlinear goniometric equation, it is necessary to use the numerical *fsolve* solver.

$$\mathbf{t}_{sHb} \cdot \mathbf{n}_{Hb} = 0 \Rightarrow \rho_k \tag{20}$$

6 Geometry of pinion tooth flank

The surface of the pinion tooth flank can be calculated using the equation of rack tooth flank surface (16) and contact angle ρ_k:

$$\mathbf{s}_{Kb} = \mathbf{s}_{Hb}(k, z, \rho_k) \tag{21}$$

The figure 4 below is an example of a solution created using the presented approach. The number of teeth z is 24, modulus m 3 mm, angle β 15°.

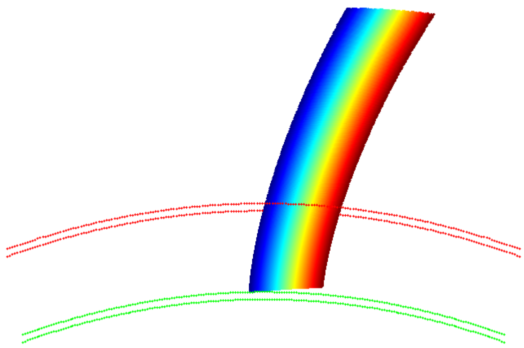

Figure 3: Calculated pinion tooth flank surface, red arcs - pitch circle, green arcs - base circle

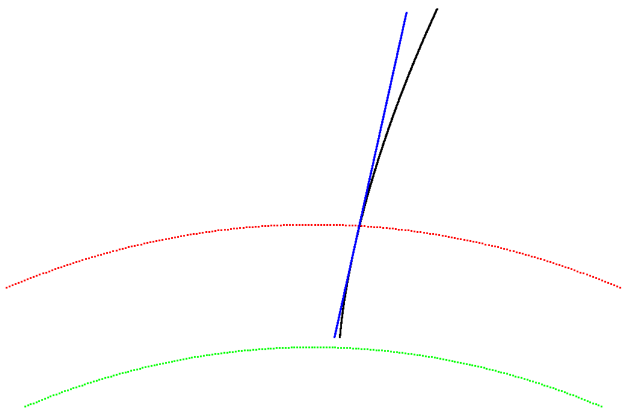

Figure 4: Contact of rack (blue) and pinion (black) profile, red arc - pitch circle, green arc - base circle

7 Conclusion

Creation of geometry based on this contact theorem is one way to create the geometry of the mating gear. It is relatively simple, but requires a numerical solver.

Acknowledgments

The research work reported here was made possible by project SGS-2016-012.

References

[1] CSN 01 4607. *Ozubena kola celni s evolventnim ozubenim - Zakladni profils*. Praha: Urad pro normalizaci a mereni, 1978.

Martin Kratochvil
Department of Machine Design, University of West Bohemia, Univerzitni 2732/8, 301 00 Pilsen, Czech Republic, kratochv@kks.zcu.cz