# DESIGN OF CONTINUOUS LEAD COMPENSATORS

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**Abstract**

**In this paper design of continuous lead compensators with specified structure for linear system is addressed, proposed and tested. The design of lead compensators is proposed by using Bode diagram. The paper deals with theoretical and practical methodology, and its successful application for controlled systems. Advantages and disadvantages of phase lead compensators are summarized.**

## 1 Introduction

Automation algorithms are installed in various areas of industrial production equipment, which allow self-control of processes without the involvement of humans. One of such devices is the correcting element with specified structure [1]. The phase lead compensator with specified structure is a dynamic system, which adjusts static and dynamic characteristics of controlled system. In this paper serial connection of above mentioned controllers is discussed, see Fig. 1.

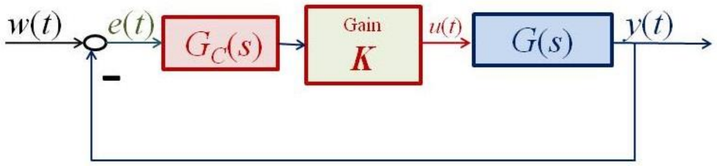


Figure 1: Control structure with compensator $G_C(s)$ – $w(t)$ is reference input signal, $e(t)$ is error signal, $u(t)$ is input signal to the plant model with transfer function $G(s)$, $y(t)$ is plant output

The simplest structure of the phase lead compensator is the first order system with transfer function

$$G_C(s) = \frac{aTs+1}{Ts+1} \tag{1}$$

where $K, T, a$ computed parameters, $a$>1. The zero is always located to the right of the pole, and the distance between them is determined by the constant $a$.

## 2 Design of Phase Lead Compensator

Stability or speed of a system response can be increased by the phase lead compensator. It is typically used for servos. By using this kind of compensator is moved the phase $\varphi(\omega)$ for angular frequency $\omega \in \langle 0, \infty \rangle$ into the positive values of the range of interval $\varphi(\omega) \in \langle 0, \pi/2 \rangle$. The transfer function of compensator is given by (1). Bode diagram for continuous phase lead compensator is shown in Fig. 2.

Maximum value of the phase $\varphi_m$ is reached at a angular frequency $\omega_m$, which is the center of interval that is bounded with two corner points $1/aT$ and $1/T$ of magnitude curve.

$$\log \omega_m = \frac{1}{2}\left[\log\frac{1}{aT} + \log\frac{1}{T}\right] \tag{2}$$

from where

$$\omega_m = \frac{1}{T\sqrt{a}} \tag{3}$$

and

$$T = \frac{1}{\omega_m \sqrt{a}} \tag{4}$$

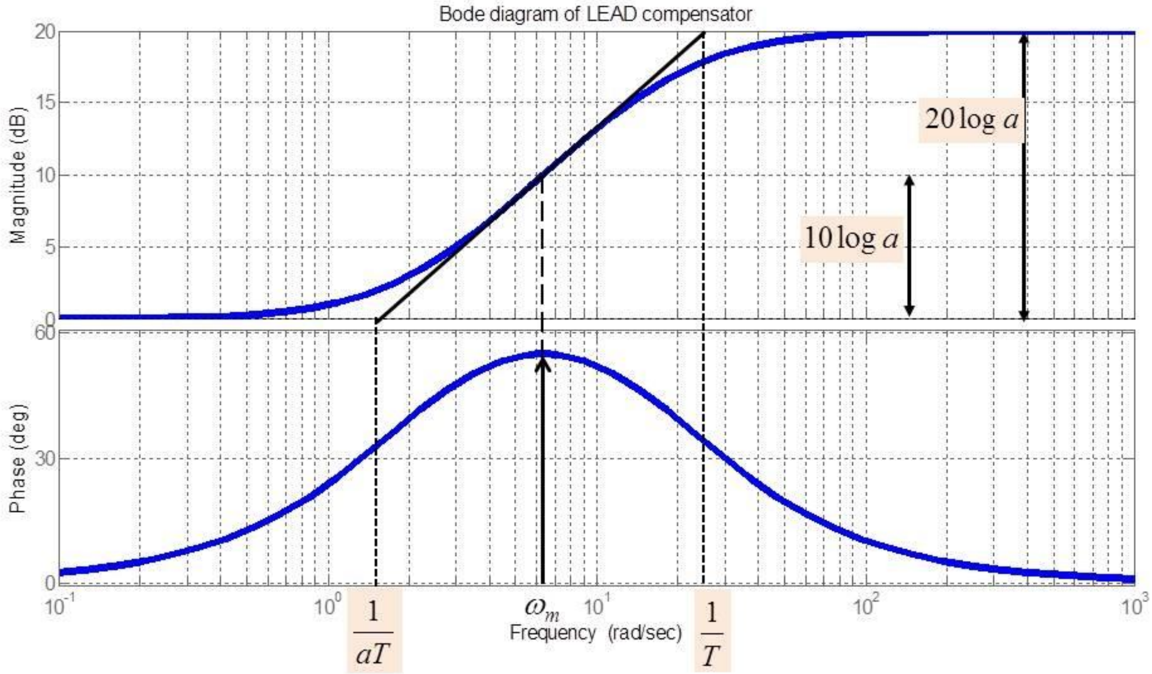


Figure 2: Bode diagram of phase lead compensator

Form frequency transfer function of compensator

$$G_C(j\omega) = \frac{aT^2\omega^2 + 1}{T^2\omega^2 + 1} + j\frac{T(a-1)\omega}{T^2\omega^2 + 1} = P(\omega) + jQ(\omega) \tag{5}$$

is expressed the maximum phase

$$\sin\varphi_m = \frac{tg\varphi_m}{\sqrt{tg^2\varphi_m + 1}} = \frac{a-1}{a+1} \tag{6}$$

$$\lim_{\omega\to\infty}\left[20\log|G_C(j\omega)|\right] = 20\log a \tag{7}$$

The parameter $a$ can be derived from equation (6) as follows

$$a = \frac{1+\sin\varphi_m}{1-\sin\varphi_m} \tag{8}$$

Obtained parameter $a$ is used for evaluation the equation (7) and for $\omega_m$ holds the relationship

$$20\log|G_C(j\omega)| = 10\log a \tag{9}$$

By using the phase lead compensator is moved the magnitude curve in point $\omega_m$ of value 10log $a$ towards positive values. If the angular frequency $\omega_m$ has to be the amplitude intersection of an magnitude curve of an open-loop system with a compensator, $\omega_m$ is looked for on uncompensated magnitude characteristic for amount 10log $a$.

Algorithm of design consists of the following steps [2]:

1. Determining of the value of gain $K$ for open control loop. It is chosen to satisfy steady-state-error requirement.
2. For expressed value of the gain is constructed the Bode diagram of uncompensated open control loop $KG(s)$. From Bode diagram is read value of phase margin $\Delta\varphi_{O1}$. Then $\varphi_m$ is calculated from (10).

$$\varphi_m = \Delta\varphi_{OZ} - \Delta\varphi_{O1} \tag{10}$$

where $\Delta\varphi_{O1}$ is phase margin determined from Bode diagram, $\Delta\varphi_{OZ}$ is required value of phase margin.
The magnitude intersection is moved by phase lead compensator towards higher values of $\omega$, where $\Delta\varphi_{O1}$ is has less positive (or more negative) value and the value of $\varphi_m$ is increased according equation (10). This growth is estimated in the range of 5°-15°. So $\varphi_m$ must be increased by this estimate.

3. For desired value $\varphi_m$ is calculated the parameter $a$ from equation (8).
4. The frequency $\omega_m$ is where the magnitude of the uncompensated $KG(j\omega)$ is -10log $a$ dB.
5. The constant $T$ can be determined from equation (4). Design of phase lead compensator is now completed.
6. Check the design by simulating the step response. Plot also the Bode diagram of compensated open control loop. If all performance requirements are met, stop. Otherwise go back to step 2 and value of $\varphi_m$ must be chosen.

# 3 Advantages and disadvantages of phase lead compensator

Advantages are [3]:
- the phase margin is improved,
- an improvement of relative stability of the system and damping,
- the bandwidth of the closed-loop system is increased,
- the reducing of maximal overshoot and rise time.

Disadvantages are:
- if uncompensated control circuit is unstable, phase uplift $\varphi_m$ may be close to the 90°or may exceed it,
- if $\varphi_m$ is close to 90°, calculation leads to a large value of the parameter $a$, which can cause noise amplification,
- if $\varphi_m$ is higher than 90°, is needed to involve more phase lead compensators into the series and transfer function of the compensator is then

$$G_C(s) = \left[\frac{aTs+1}{Ts+1}\right]^n \qquad n = 2,\ 3,\ \ldots \tag{11}$$

# 4 Case Study and Simulation Results

Consider the following system

$$G(s) = \frac{1}{s(T_1 s+1)(T_2 s+1)} = \frac{1}{s(s+1)(0.0125s+1)} \tag{12}$$

The task is to design a continuous compensator that satisfies the following performance requirements:

1. Steady-state error $e(\infty) < \varepsilon$ due to unit step fuction input $w(t)$=1, where $\varepsilon$=0.01.
2. Maximum percent overshoot $\eta_{\max}$<30% or phase margin of the system $\Delta\varphi_{OZ}$ >45°.

Solution of the example:

The gain $K$=100 is expressed from the steady-state error for $w(t)$=1 as follows

$$\begin{gathered} e(\infty) = \lim_{s\to 0} sE(s) = \lim_{s\to 0} s\frac{1}{1+G_0(s)}W(s) \\ e(\infty) = \lim_{s\to 0} s\frac{W(s)}{1+KG(s)} = \lim_{s\to 0} s\frac{\frac{1}{s}}{1+K\frac{1}{s(s+1)(0.0125s+1)}} = \frac{1}{K} \le 0.01 \Rightarrow \quad K \ge 100 \end{gathered} \tag{13}$$

The controller is now a $P$ (proportional) controller. The transfer function of open-loop system will be as follows

$$G_0(s) = KG(s) = \frac{100}{s(s+1)(0.0125s+1)} \tag{14}$$

Bode diagram for $G_0(s)$ is shown in Fig. 3.

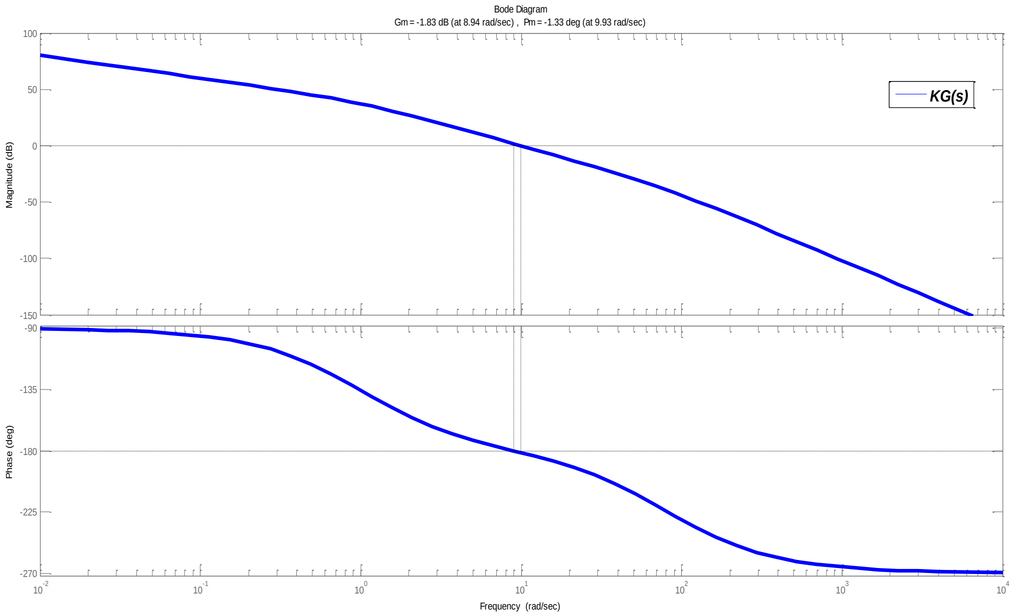


Figure 3: Bode diagram of (14)

The uncompensated gain crossover frequency (with $K$) is 9.93 rad/sec and the phase margin is $\Delta\varphi_{OZ}$ =-1.33°.

The value of $\varphi_m$ can be expressed according to equation (10), and thus $\varphi_m$=45°-$\Delta\varphi_{OZ}$, so $\varphi_m$=45°+1.33°=46.33°. So $\varphi_m$ must be increased. So it has been chosen $\varphi_m$=55°. As shown, this value is going up to 8.67°.

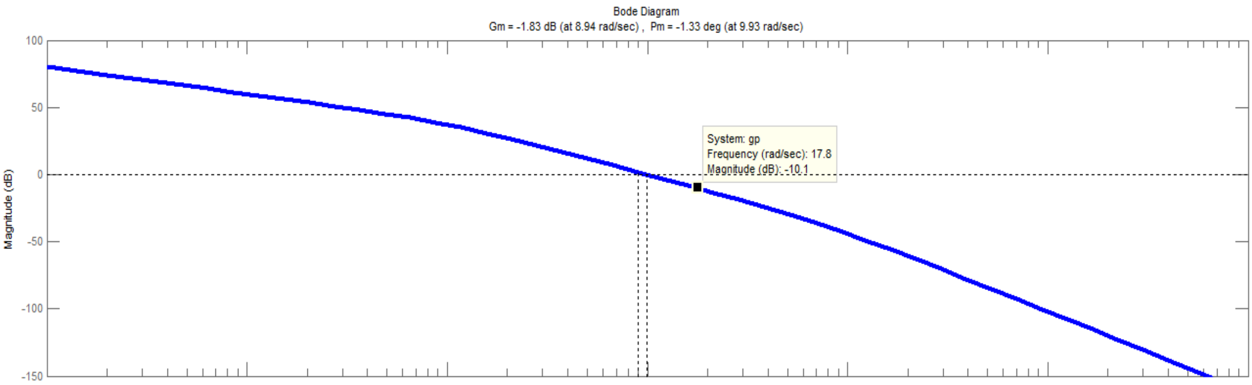


Figure 4: Bode diagram – amplitude plot of (14)

The parameter $a$=10 according to equation (8). The magnitude curve will be shifted by 10log $a$=10log(10)=10 dB. The compensated gain crossover frequency $\omega_m$ will be deducted from Bode diagram at the magnitude $|KG(j\omega)| = -10$ dB, see Fig. 4. Therefore, the frequency $\omega_m$=17.8 rad/sec. The parameter $T$=0.018 can be expressed from (4).

The final phase lead compensator (1) is

$$G_C(s) = \frac{0.18s+1}{0.018s+1} \tag{15}$$

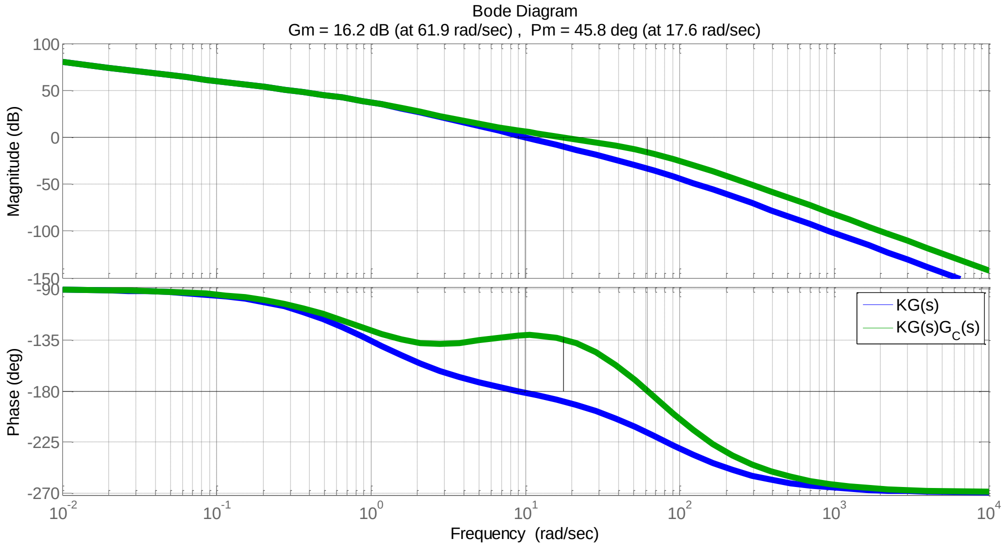


Figure 5: Bode diagram for compensated (green) and uncompensated system (blue)

In Fig. 5 the blue curve are for uncompensated system and green for system with the phase lead compensator. The compensated gain crossover frequency of 17.6 rad/sec and the phase margin Pm=45.8°. Time response of the output variable with lead compensator is shown in Fig. 6.

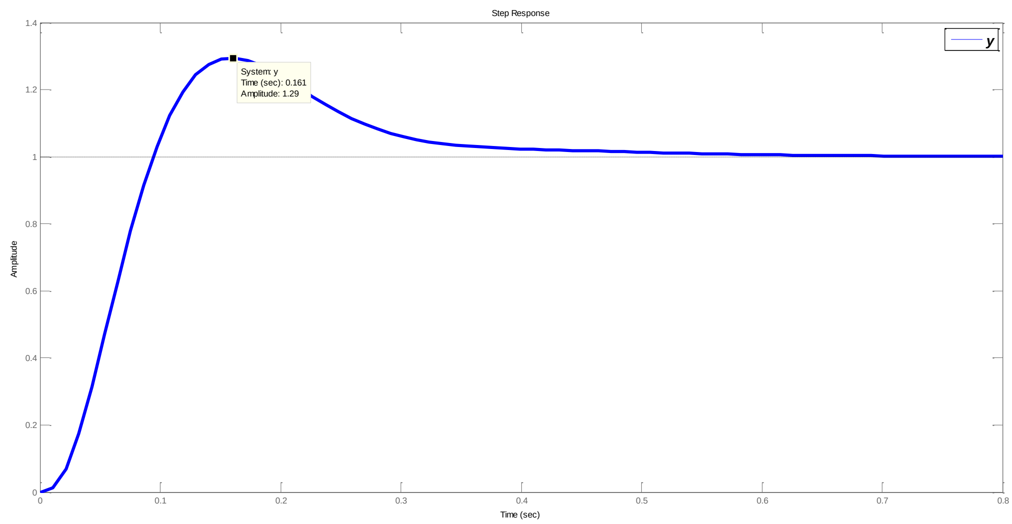


Figure 6: Time responses of the output variable with lead compensator

Maximum percent overshoot $\eta_{max}$=29%.

## 5 Conclusions

The specifications of the system (12) were satisfied for compensator the phase lead (15). The phase margin, steady-state error and maximal percent overshoot specifications were satisfied. As shown in figures, the phase lead compensator increases the gain of the system at high frequencies. This can increase the crossover frequency, which will help to decrease the rise time and settling time of the system.

**ACKNOWLEDGEMENT**

This paper has been supported by the Slovak Scientific Grant Agency, Grant No. 1/2256/12.

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