DESIGN OF ROBUST MIMO CONTROL USING GENETIC ALGORITHM

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Abstract

The paper deals with the robust controller design using genetic algorithm for uncertain MIMO systems. The genetic algorithm is based on optimisation procedure, where the cost function to be minimized comprises the closed-loop simulation of the controlled MIMO process and evaluation of a selected performance index. Using this approach, the PID controller parameters were optimised to obtain the required behaviour of the controller process. Practical implementation and comparison of the proposed genetic method with conventional approaches (Small Gain Theory, Equivalent Subsystems Method) are verified and tested on a real process – two interconnected DC motors with varying parameters.

1 Introduction

Present trends in the design complex process control require an increasing degree of integration of numerical mathematics, control engineering methods, new control structures based on distribution, embedded network control structure and new information and communication technologies. Furthermore, increasing problems with interactions, process nonlinearities, operating constraints, time delays, uncertainties, and significant dead-times lead to the necessity to develop more sophisticated and robust control strategies.

Main ideas covered in this paper are motivated by the development of new advanced robust control approaches and new possibilities of their controller structures implementation.

For real MIMO plants, a controller design has to cope with the effect of uncertainties, which very often cause a poor performance or even instability of closed-loop systems. The reason for that is a perpetual time change of parameters (due to aging, influence of environment, working point changes *etc.*), as well as unmodelled dynamics. The former uncertainty type is denoted as a parametric uncertainty and the latter one a dynamic uncertainty. A controller guaranteeing closed-loop stability under both of these uncertainty types is a robust controller. Since many problems in robustness analysis and synthesis can be formulated as minimization of a cost function with respect to controller parameters, creative combinations of a variety of pre-existing control methodologies and the genetic approach will result in powerful tools that can address real engineering control problems.

The paper presents genetic approaches to robust PID controller design for a laboratory plant consisting of two interconnected DC motors with varying parameters. The designed approach uses a genetic algorithm applied for the affine model. The genetic algorithm represents an optimization procedure, where the cost function to be minimized comprises the closed-loop simulation of the controlled process and a selected performance index evaluation [2]. Using this approach, the parameters of the PID controller were optimised in order to obtain the required behaviour of the controlled process [1, 8, 9]. Comparison of the proposed genetic method with conventional approaches (Small Gain Theory [11, 12], Equivalent Subsystems Method [7]) are verified and tested on a real process – two interconnected DC motors with varying parameters.

2 Robust controller design

2.1 Preliminaries

Consider a MIMO system described by a transfer function matrix $G(s) \in R^{m \times m}$, and a diagonal controller $R(s) \in R^{m \times m}$ in the standard feedback loop (Fig. 1), where *w, u, y, e* are vectors of reference, control, output and control error, respectively, of compatible dimensions.

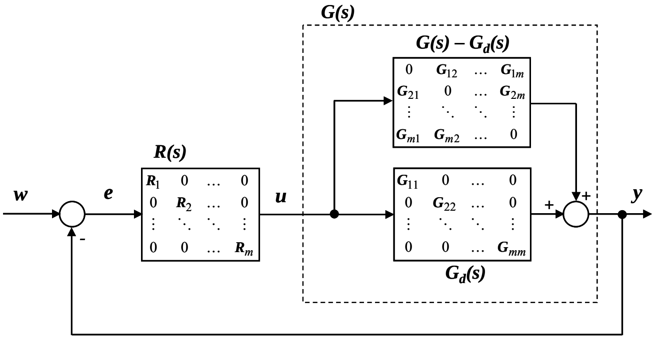

Figure 1: Feedback loop under decentralized controller

Let the uncertain plant model be given as a set of N transfer function matrices in N different operating points

$$G^k(s) = \left\{ G_{ij}^k(s) = \frac{B_{ij}^k}{A_{ij}^k} \right\}_{m \times m}, \quad k = 1,2,\ldots,N \tag{1}$$

with $G_{ij}^k(s) = \frac{y_i^k(s)}{u_j^k(s)}$, $i, j = 1,2,\ldots,N$, where $y_i^k(s)$ is the i-th output and $u_j^k(s)$ is the j-th input of the plant in the k-th experiment.

Consider a MIMO system (1) and let the set of the *l-th* subsystem parameters of the controlled MIMO system be

$$s_l = \{s_{l1}, s_{l2}, \ldots, s_{lp}\} = \{b_{lr}, \ldots, b_{l0,} a_{lr}, \ldots, a_{l1}\} \tag{2}$$

During the plant operation, the parameters s_l can vary within some uncertainty domain for each $G_{ij}(s)$ for the minimum and maximum possible values of the i-th subsystem parameter, respectively.

Consider $c_j = \{c_{j1}, \ldots, c_{jq}\} = \{P_{j,} I_{j,} D_j\}$ to be the set of designed j-th PID controller parameters, where $j = 1,\ldots,m$. For N different working points of the controlled process, defined by different vectors s_l which are to be controlled by the robust controller.

2.2 Robust controller design for MIMO system using the Genetic Algorithm

For this case, which is shown in Figure 1 and defined equations (1) and (2), consider the cost function in the additive form

$$J = \sum_{k=1}^{N} J_i \tag{3}$$

comprising performance evaluation in all N working points. It is also recommended to include the measured noise from the real system or other possible disturbances or expected situations in the simulation model.

The controller design principle is actually an optimization task - search for such controller parameters from the defined parameter space which minimize the performance index. The cost function (fitness) is a mapping $R^{m.q} \to R$, where m is the number of system inputs (number of controllers) and q is the number of designed controller parameters. The cost function can represent a sum of absolute control errors (SAE) in the following form:

$$J = \sum_{j=1}^{m} \sum_{i=1}^{N_y} \left| e_{ji} \right| = \sum_{j=1}^{m} \sum_{i=1}^{N_y} \left| w_{ji} - y_{ji} \right| \tag{4}$$

where w_j is reference variable, y_j is controlled output, e_j is control error, m is number of system outputs and N_y is number of patterns. Fitness is represented by a cost function, or in the case of control by a modified cost function which can include e.g. penalized derivation of process output y_j and/or derivation of control action u_j. The modified cost function is in the following form:

$$J = \sum_{j=1}^{m}\sum_{i=1}^{N_y}\left|e_{ji}\right| + \alpha.\sum_{j=1}^{m}\sum_{i=1}^{N_y}\left|\frac{dy_{ji}}{dt}\right| + \beta.\sum_{j=1}^{m}\sum_{i=1}^{N_y}\left|\frac{du_{ji}}{dt}\right| \tag{5}$$

where α, β are weight constants.

Genetic algorithms are described in e.g. [2, 4, 6, 8, 9, 10] and others. Each chromosome represents a potential solution, which is a linear string of numbers, whose items (genes) represent in our case the designed controller parameters. Without loss of generality let us consider a PID controller with feedforward structure where P_j, I_j, D_j are controller parameters of the *j-th* PID controller and t is time.

$$u_j(t) = P_j e_j(t) + I_j \int_0^t e_j(t)dt + D_j \frac{de_j(t)}{dt}, \quad j = 1,...,m \tag{6}$$

The searched PID controller parameters are $P \in R^+$, $I \in R^+$, $D \in R^+$. In this case the chromosome representation can be in the form $\mathbf{ch}=\{P_1,I_1,D_{1, \ldots,} P_j,I_j,D_{j,\ldots,} P_m,I_m,D_m\}$.

A general scheme of a GA has the following steps (Figure 2):

1. Initialisation of the population of chromosomes (set of randomly generated chromosomes).

2. Evaluation of the cost function (fitness) for all chromosomes.

3. Selection of parent chromosomes.

4. Crossover and mutation of the parents → children.

5. Completion of the new population from the new children and selected members of the old population. Jump to the step 2.

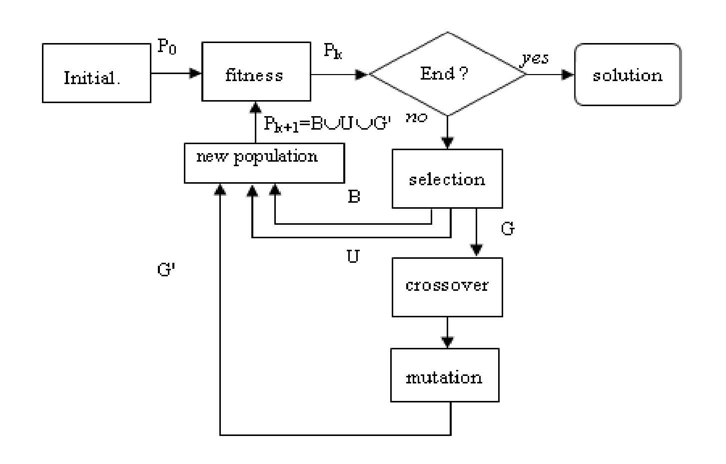

Figure 2: Block scheme of the used genetic algorithm

A block scheme of a GA-based design is in Figure 3. Before each cost function evaluation, the corresponding chromosome (genotype) is decoded into controller parameters of the simulation model (phenotype) and after the simulation the performance index is evaluated.

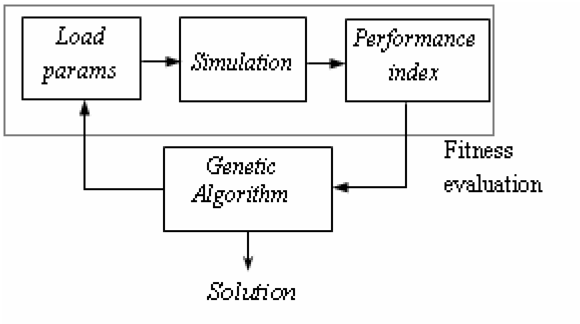

Figure 3: Block scheme of the GA-based controller design

3 Case study

3.1 The laboratory MIMO system of DC-motors

The laboratory MIMO plant consisting of two DC-motors with interconnections between them across filters is shown in Figure 4.

Mechanical interconnection is realized using inertia load and spring. Power supply, measurement of signals and motor control are supplied by motor electronics. In electronics, there is a RC component connected to the motor input to enable changing the time constant and the gain of the controlled system. System dynamics parameters can be tuned with a potentiometer. The measured voltage values of potentiometers are variables load z_1 and z_2. The motor load ranges within the interval 0 – 10 [V]. The motor input variables u_1 and u_2 range within the interval 0 – 10 [V]. The controlled variables y_1 and y_2 are angular velocities sensed by optical electronic and transformed to the output voltage in the range 0 – 10 [V]. Interconnection of the controlled process with the computer, and the software LABREG for identification and control of the real plant [3] are realized through the Advantech data acquisition card of type PCI 1711.

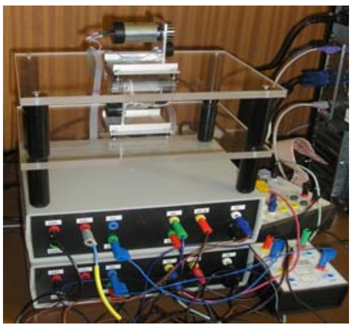
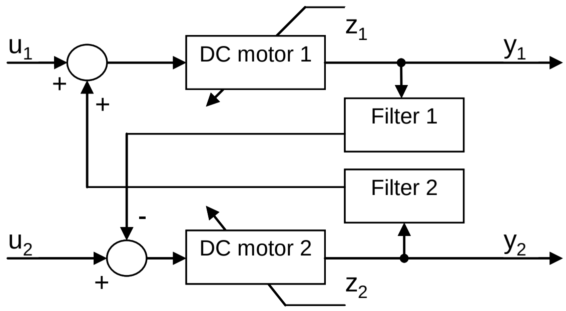

Figure 4: The laboratory MIMO system and block scheme of DC motors

3.2 Identification of DC-motors in working points

The focus of this paper is on robust PID controller design to control speed in the selected area of the MIMO system specified by several working points. The identified models are used to design robust controllers and simulate the performance index in the genetic algorithm. Performance of the designed PID robust controllers is compared on the plant in several working points. To design robust controllers, identified models of MIMO system in three working points have been used.

Consider the subsystem transfer functions of the MIMO plant (1) in the following form:

$$G_{ij}^{k}(s)=\frac{K}{a_2 s^2+a_1 s+1}e^{-T_d s} \tag{7}$$

Consider the entries of the MIMO transfer function matrix obtained by identification in three various settings of the varying loads z_1 and z_2 (voltage in potentiometer) defining three working points:

WP1: z_1=1 [V], z_2=1 [V]

WP2: z_1=5 [V], z_2=5 [V]

WP3: z_1=9 [V], z_2=9 [V]

Coefficients of the individual transfer functions in Table 1 were obtained using genetic algorithm, where the criterion function was the sum of square errors between the outputs of the plant and the identified model.

TABLE 1: COEFFICIENTS OF TRANSFER FUNCTIONS OF DC MOTORS

WP	subsystem	K	a_2	a_1	T_d
WP1	G_{11}	0.9812	0.3519	1.0469	0
	G_{22}	0.8773	0.1653	0.8315	0
	G_{12}	-0.1257	0.9999	1.4917	0.7
	G_{21}	0.1057	0.8499	1.2872	0.5
WP2	G_{11}	0.9054	1.0301	1.9773	0
	G_{22}	0.7900	0.6699	1.7179	0
	G_{12}	-0.0931	4.1600	3.2547	0.7
	G_{21}	0.09	3.1125	2.8531	1
WP3	G_{11}	0.8345	1.0712	2.6930	0
	G_{22}	0.7110	0.5994	2.5119	0
	G_{12}	-0.0681	0.1606	4.1404	1.4
	G_{21}	0.0657	9.4037	4.6146	1.5

3.3 Robust controller design using Genetic algorithm

A simulation scheme consisting of two PID controllers and the MIMO plant model in three working points was used. To evaluate the performance index (5) in all working points. Robust PID controllers have been designed by genetic algorithm for the MIMO system model according to Table 1. Following controller parameter ranges have been selected $P_k = \langle 0;5 \rangle$, $I_k = \langle 0;5 \rangle$ and $D_k = \langle 0;5 \rangle$. Optimal values of PID controller parameters obtained using GA in the user-defined search space are

$$\begin{aligned} R_1(s) &= 1.0498 + \frac{0.6275}{s} + 0.2454s \\ R_2(s) &= 1.2946 + \frac{0.8257}{s} + 0.4684s \end{aligned} \tag{8}$$

For optimal relationship between performance criteria (overshoot, settling time), weights in the performance index (5) were adjusted to α=0, β=1. In Table 2, performance indices for the three considered methods (GA – Genetic Algorithm, SGT - Small Gain Theory, ESM - Equivalent Subsystems Method) in all working points are computed; for testing, boundary distribution of working points was used where the range of the varying load as $z_k = \langle 1; 9 \rangle$ has been selected. The sum of absolute control errors (SAE) according (4), average values of overshoot and settling time are compared as well [5].

TABLE 2: PERFORMANCE INDICES FOR SELECTED METHODS

Method	Output	SAE	Average values of overshoot [%]	Average values of settling time [s]
GA	y_1	91.68	2.27	8.1
	y_2	82.75	4.36	8.6
SGT	y_1	99.67	1.49	9.4
	y_2	89.85	2.69	9.8
ESM	y_1	102.86	2.91	8.7
	y_2	101.85	5.17	10.6

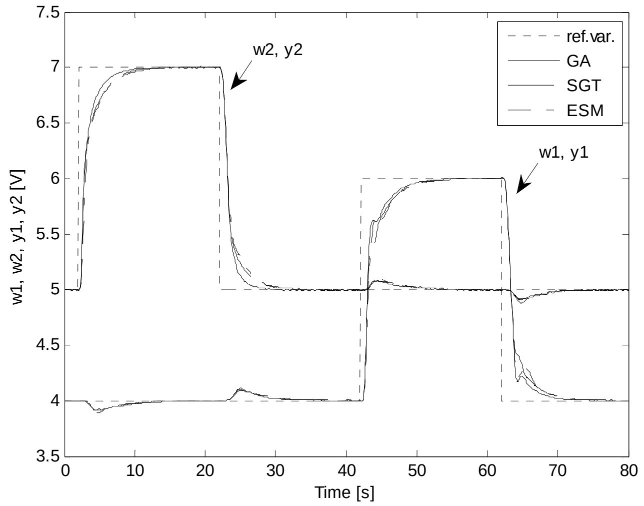

Figure 5: Closed-loop step responses in WP1

Closed-loop time responses of the reference and controlled variables for DC motors measured in the first working point (WP1) are depicted in Fig. 5, and control variables are depicted in Fig. 6. Details of time responses are depicted in Fig. 7. - 9.

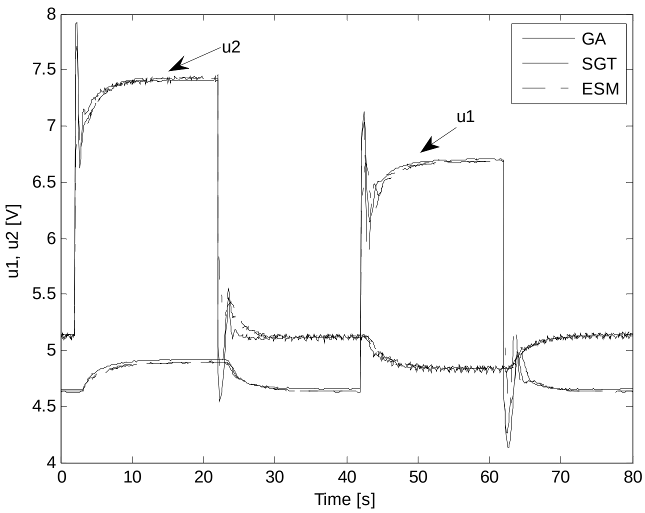

Figure 6: Time responses of control variables in the WP1

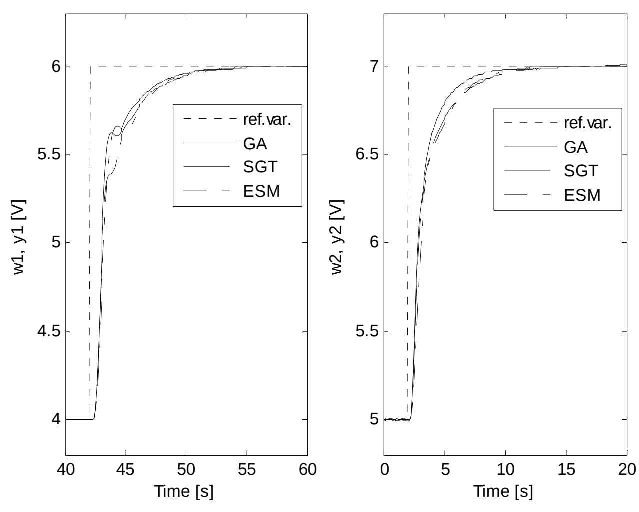

Figure 7: Details of closed-loop step responses in the WP1

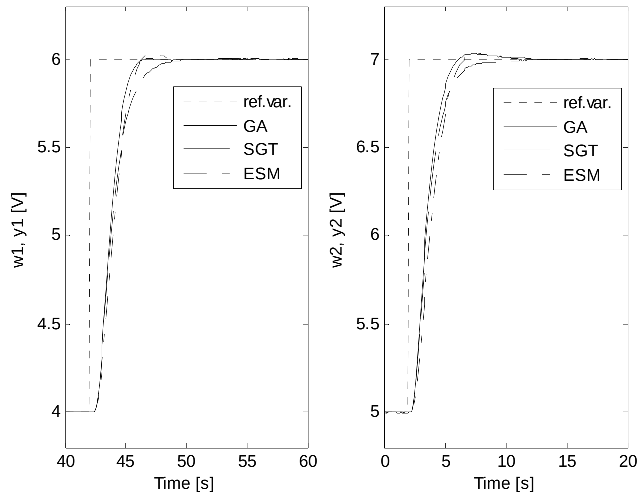

Figure 8: Details of closed-loop step responses in the WP2

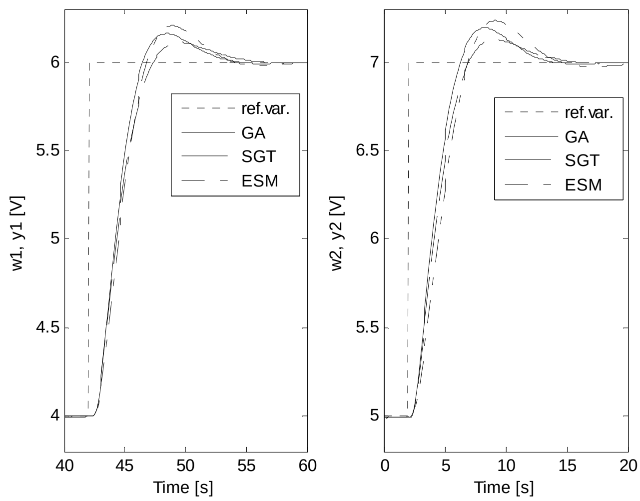

Figure 9: Detail of closed-loop step responses in the WP3

4 Conclusion

Development of efficient robust controller design approach for MIMO plants is a significant and challenging task. In this paper, three robust decentralized control design methodologies are proposed and compared (SGT, EQS and GA). The presented methods guarantee robust stability and optimal setting of controller parameters. The results obtained by verification on the real plant consisting of interconnected DC motors with varying load show the effectiveness of the proposed methods. In the robust genetic controller design the weights in the performance index have physical meaning: they can be used to adjust performance parameters in terms of overshoot and settling time. Robustness of the designed decentralized controller is investigated with respect to both stability and performance.

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