SURFACE OF CONSTANT PRECISION IN A SPECIFIC DIRECTION (VISUALIZATION IN MATLAB)

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Abstract

The example of surface of constant precision shows that graphic representation of complex space objects allows for disclosure of their basic properties.

1 Introduction

The aim of the article is to approximate demonstratively the surface that is three-dimensional equivalent of Helmert's curve and compare this surface (hereinafter referred to as the Helmert's surface) with the standard error ellipsoid.

The numerical values in the specified cases are fictitious and dimensionless values (without physical dimension). Their units depend on the nature of random vector – for the most typical case of three rectangular coordinates it would be metres or millimetres.

2 Helmert's surface

Accuracy of point positioning in the space (3D) is given by the covariance matrix Σ (3,3) and in graph it is depicted by standard error ellipsoid. The error ellipsoid characterizes position accuracy, but the ellipsoid vector does not set accuracy in a specific direction. If it is the aim to determine accuracy in a specific direction, e.g. when monitoring shift, it is necessary to use the surface (3D analogy to the Helmert's curve) instead of an error ellipsoid. This surface is closely related with the error ellipsoid – it is defined by the same covariance matrix, it has the same semi-axes. The behaviour of this surface is given by the parametric equation

$$r_H^2 = a^2 \cos^2 \alpha \ \cos^2 \beta + b^2 \sin^2 \alpha \ \cos^2 \beta + c^2 \sin^2 \beta ,$$

where r_H, α and β are space polar coordinates.

Surface shape and properties are not directly apparent from the equation. Therefore, it is to advantage to plot (visualize) the surface in general form $a \neq b \neq c$ also for special cases, such as $a = b = c$, $a = b \neq c$ etc., to reveal its basic features and relation to the error ellipsoid. The MATLAB software was used for visualization allowing for both descriptive drafting and handling (e.g. rotation) with the image and, last but not least, numerical calculations to find out basic surface characteristics.

The shape and properties of the surface are not directly traceable from the equation. It is therefore possible to draw the surface (to visualize it).

3 Results

The figure 1 shows three-axis ellipsoid with semi-axes $a = 7$, $b = 4$, $c = 2$. Helmert's surface corresponding to this ellipsoid is shown in the figure 2. The differences of both surfaces are apparent from sections in the figure 3 and 4. Similarly as in case of ellipse and Helmert's curve also in case of ellipsoid and Helmert's surface the radius vectors in the semi-axle directions are the same and in general direction the radius vector of the Helmert's surface is bigger than the ellipsoid radius vector. The difference of radius vectors depends on the sizes of semi-axles and on the α, β angle values. Figure 5 shows the behaviour of the differences for the above semi-axles, i.e. for the ellipsoid from figure 1 and surface from figure 2.

Figure 6 displays the behaviour of the ratios of radius vectors of the Helmert's surface and error ellipsoid for the semi-axles $a = 7$, $b = 4$, $c = 2$.

Figure 2 represents an example of the Helmert's surface pertaining to the three-axis ellipsoid, but also special shapes of Helmert's surfaces correspond to special cases of ellipsoid. For $a = b = c$ the Helmert's surface is identified with the error ball, for $a \neq b = c$ and for $a = b \neq c$ the rotational surfaces will be formed pertaining to oblate and oblong ellipsoid. The figures 7 and 8 are the examples for $a = 7$, $b = c = 2$ and for $a = b = 7$, $c = 2$.

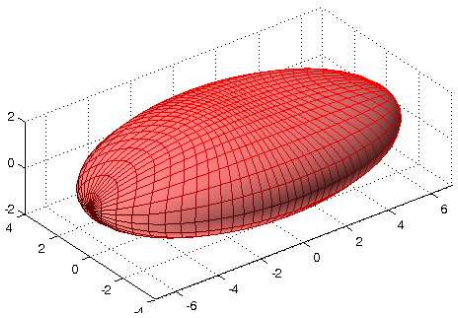

Fig. 1: Ellipsoid for $a = 7$, $b = 4$, $c = 2$

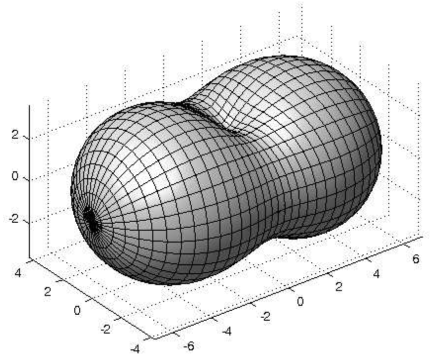

Fig. 2: Helmert's surface for $a = 7$, $b = 4$, $c = 2$

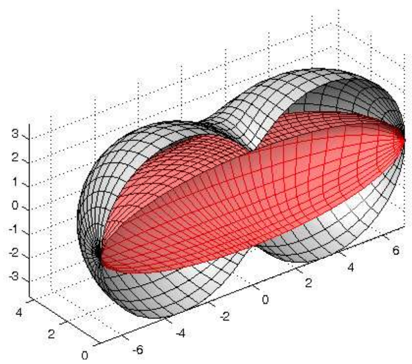

Fig. 3: Vertical section of ellipsoid and Helmert's surface

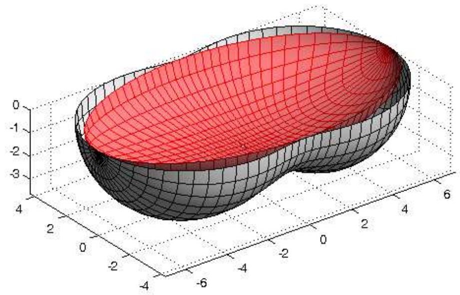

Fig. 4: Horizontal section of ellipsoid and Helmert's surface

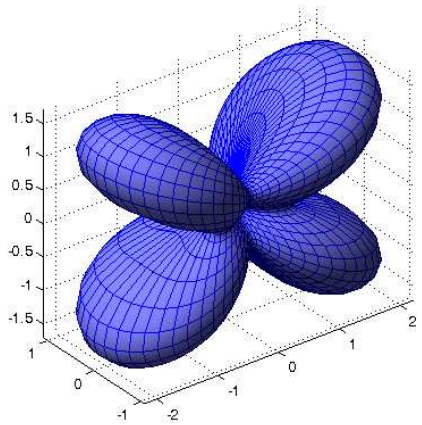

Fig. 5: Graphical illustration of difference between error ellipsoid and Helmert's surface for $a = 7$, $b = 4$, $c = 2$

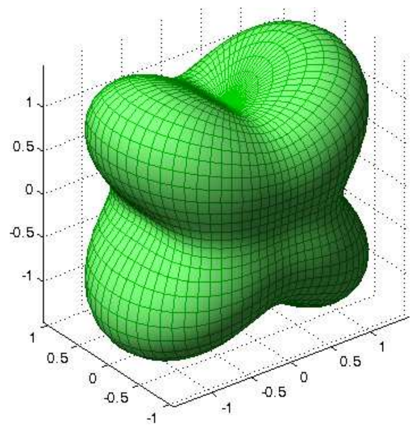

Fig. 6: Behaviour of the ratios of radius vectors of the Helmert's surface and error ellipsoid for $a = 7$, $b = 4$, $c = 2$

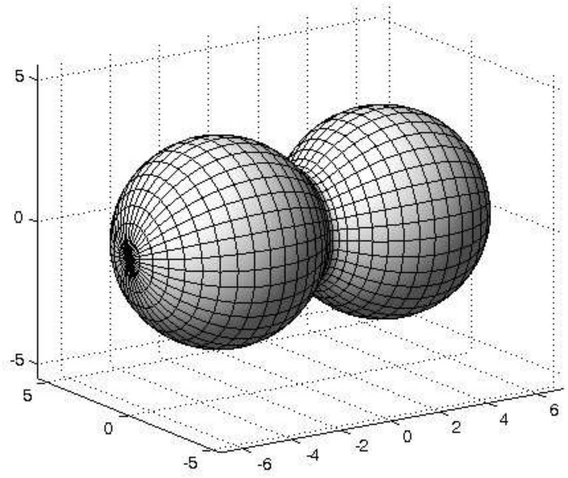

Fig. 7: Helmert’s surface for $a = 7$, $b = c = 2$

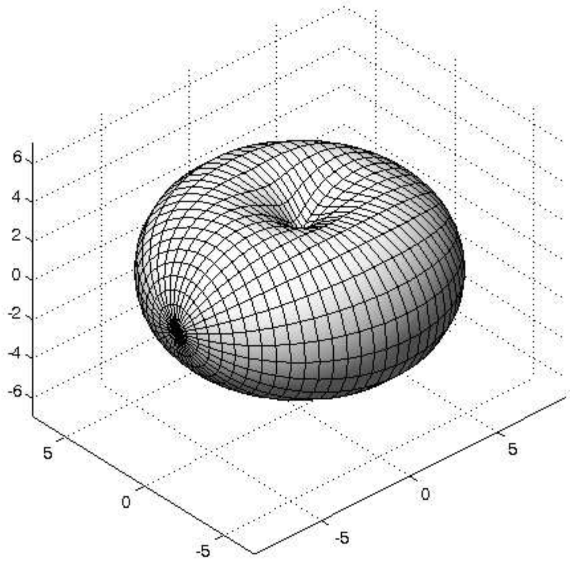

Fig. 8: Helmert’s surface for $a = b = 7$, $c = 2$

4 Conclusion

The example of Helmert’s surface shows that graphic representation of complex space objects allows for disclosure of their basic properties. These properties can then become the starting points for further theoretical study.

References

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