Eigenvalue study for the ignition of a self-sustaining discharge with COMSOL Multiphysics[1]

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Townsend criteria[2]

- Assumptions:
  1. No diffusion
  2. No photoionization
  3. Homogeneous electric field
- Two results:
  1. Non-self-sustaining discharge – only trivial solution
  2. Self-sustaining discharge – unstable trivial and stable non-trivial solution

$$\int_0^d \alpha\, dx < \ln\left(1+\frac{1}{\gamma}\right) \qquad \int_0^d \alpha\, dx \geqslant \ln\left(1+\frac{1}{\gamma}\right)$$

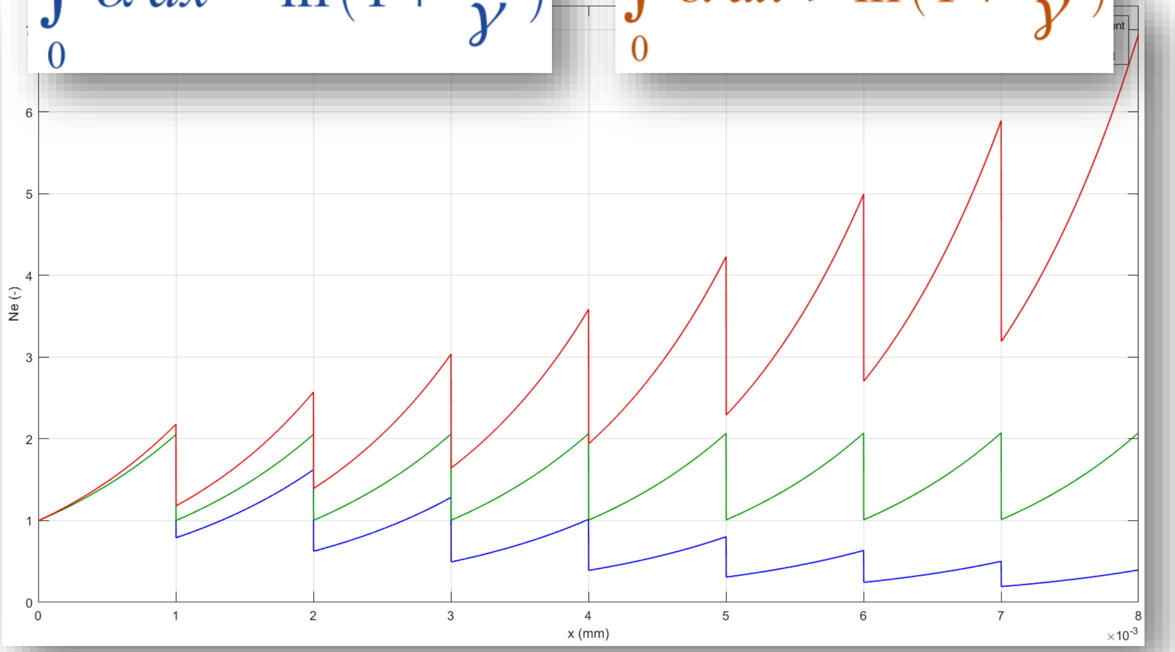


Fig. 1: Non-self-sustaining (blue) and Self-sustaining (red) discharge, critical state - green.

U → λ study – Non-trivial solutions analysis

- All voltage *U* terms in the **drift-diffusion-reaction equations** (*Coefficient Form PDE* node for electron n_e and positive ion n_p concentrations) are replaced by variable *lambda*:

$$\nabla \cdot \left\{ -D_{e,p}(E_{Td})\nabla n_{e,p} \mp \left[\mu_{e,p}\left(E_{Td,r}\right) \cdot E_r;\ \mu_{e,p}\left(E_{Td,z}\right) \cdot E_z\right] \cdot \frac{n_{e,p}}{N} \right\} - \{\alpha(E_{Td}) - \eta(E_{Td})\} \cdot N \cdot \mu_e(E_{Td}) \cdot E \cdot n_e = S_{ph}$$

- **Electric field** is defined as:

$$E = lambda \cdot es.normE$$
$$E_r = lambda \cdot es.E_r$$
$$E_z = lambda \cdot es.E_z$$
$$E_{Td} = \left| 1e21 \cdot lambda \cdot \frac{es.normE}{N} \right|$$
$$E_{Td,r} = \left| 1e21 \cdot lambda \cdot \frac{es.E_r}{N} \right|$$
$$E_{Td,z} = \left| 1e21 \cdot lambda \cdot \frac{es.E_z}{N} \right|$$

- Presented method is based on [3]

Tab. 1: Variable definitions used in the drift-diffusion-reaction-equations.

$n_{e,p}$	Electron e and positive ion p concentrations
$D_{e,p}$	Electron e and positive ion p diffusivities
$\mu_{e,p}$	Electron e and positive ion p mobilities
α	Ionization coefficient
η	Attachment coefficient
N	Neutral gas concentration
S_{ph}	Photoionization rate
E	Electric field
E_r	r-component of electric field
E_z	z-component of electric field
E_{Td}	Reduced electric field in Td (defined as E/N)
$E_{Td,r}$	r-component of reduced electric field in Td
$E_{Td,z}$	z-component of reduced electric field in Td

Geometry – Three electrode configurations in 2D axisymmetric

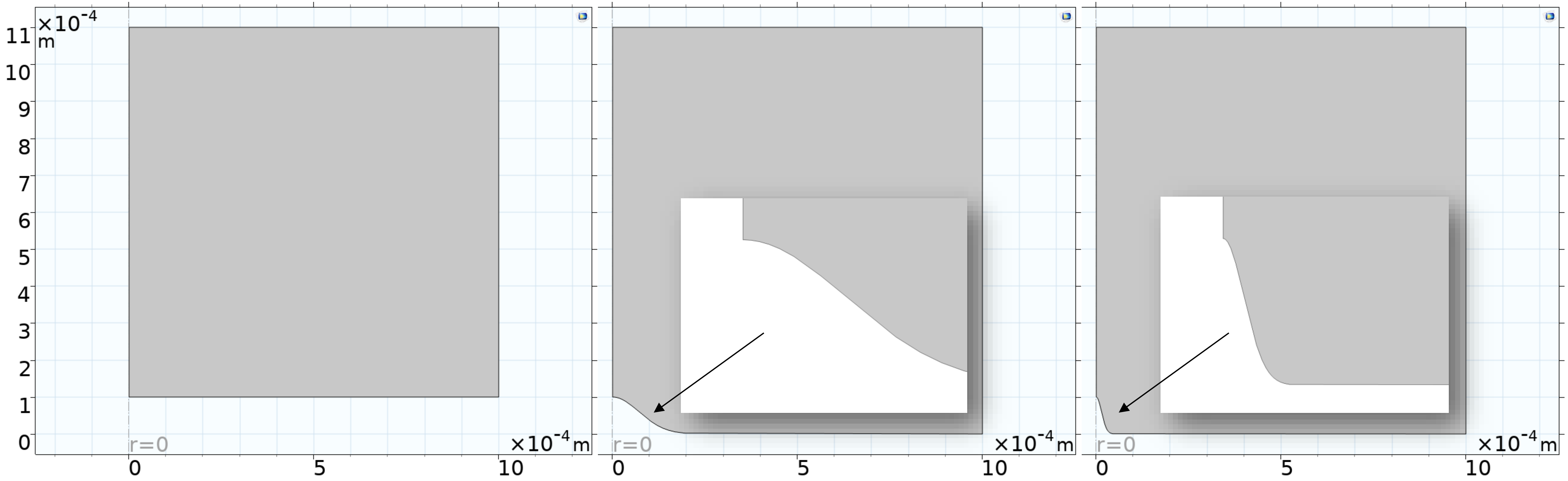


Fig. 2: Upper plane cathode (ground) against 3 different bottom anode electrodes (plane, protrusion no. 1 and protrusion no. 2, respectively). Dielectric gap distance *d* (protrusion to plane) set to 1 mm.

Boundary conditions for DDR equations

- *Flux/Source* node at the lower anode electrodes:

$$-\boldsymbol{n} \cdot (-c\nabla \boldsymbol{u} - \alpha \boldsymbol{u} + \gamma) = \mathrm{g} - \mathrm{q}\mathbf{u} \qquad \mathrm{g} = [0, 0\,]^T$$

$$\boldsymbol{u} = \left[n_e, n_p\right]^T \qquad \mathrm{q} = \begin{bmatrix} \mu_e(E_{Td}) \cdot E & 0 \\ 0 & 0 \end{bmatrix}$$

- *Flux/Source* node at the upper cathode electrode – γ in *q* matrix is secondary electron emission by positive ion-cathode interaction (set to 0.4):

$$\mathrm{g} = [0, 0\,]^T$$

$$\mathrm{q} = \begin{bmatrix} 0 & -\mu_p \cdot E \cdot \gamma \\ 0 & \mu_p \cdot E \end{bmatrix}$$

$$E_{Td} = \left| 1e21 \cdot lambda \cdot \frac{es.normE}{N} \right|$$

$$E = lambda \cdot es.normE$$

Electric field calculation

- **Space charge** does not disturb external electric field
- **External electric field** is calculated only once through *Electrostatics* physics node and then scaled by U (by λ for $U \rightarrow \lambda$ study)
- Boundary conditions are set as follows:
  - 1 V for lower three different anode electrodes
  - *Ground* for upper plane cathodes
  - *Axial symmetry* on the left side
  - *Zero charge* on the right side

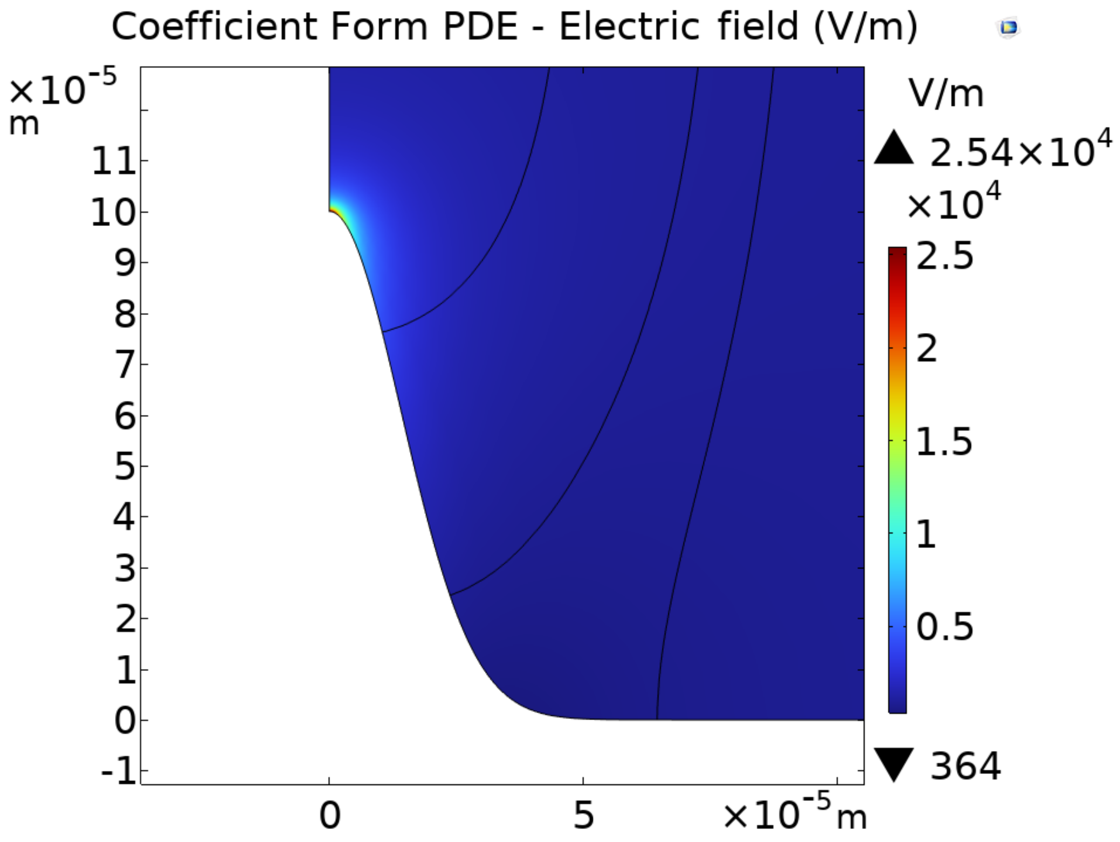


Fig. 3: External electric field calculated by *Electrostatics* physics node for protrusion no. 2.

Photoionization in air

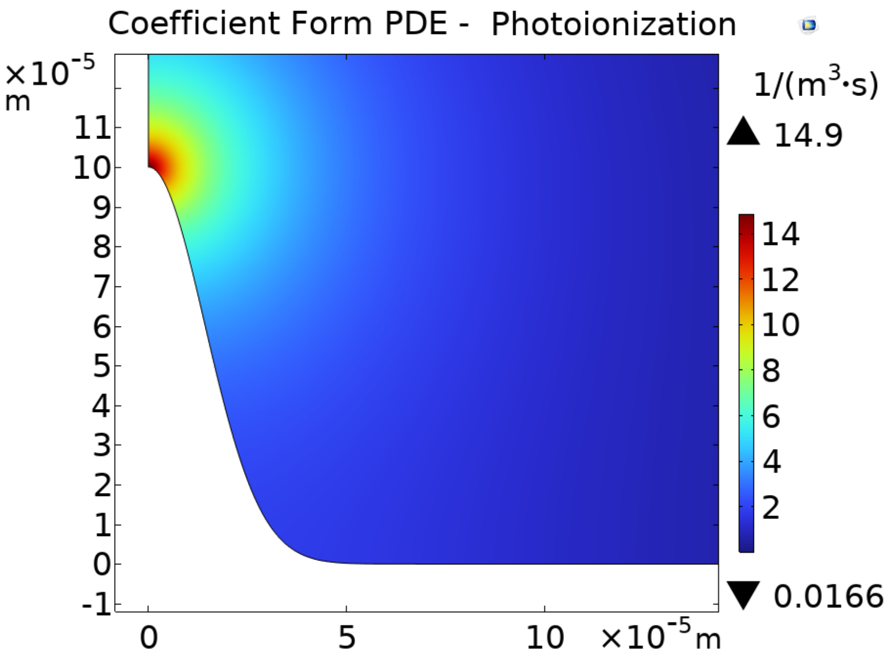


Fig. 4: Photoionization rate around protrusion no. 2.

- **Three-exponential Helmholtz** model for photoionization rate S_{ph} proposed in [4] in *Coefficient Form PDE*:

$$\Delta S_{ph,i} - (\lambda_i \cdot p_{O2})^2 \cdot S_{ph,i} = -A_i \cdot p_{O2}^2 \cdot \chi_{ext}(E_{Td}) \cdot f_q \ S_{coll}$$

$$S_{ph} = \sum_{i=1}^{3} S_{ph,i}$$

- Quenching factor f_q, partial pressure of molecular oxygen p_{O2} and parameters λ_i (not an eigenvalue) and A_i (where i = 1, 2, 3) are summarised in the *tab. 2* and collisional ionization S_{coll} is defined as follows:

$$S_{coll} = \alpha(E_{Td}) \cdot N \cdot \mu_e(E_{Td}) \cdot E \cdot n_e$$

$$E_{Td} = \left(1e21 \cdot lambda \cdot \frac{es.normE}{N}\right) \text{ and } E = lambda \cdot es.normE$$

Tab. 2: Photoionization parameters [4].

A_1	1.986e-4 [cm^-2*Torr^-2]
A_2	0.0051 [cm^-2*Torr^-2]
A_3	0.4886 [cm^-2*Torr^-2]
λ_1	0.0553 [cm^-1*Torr^-1]
λ_2	0.1460 [cm^-1*Torr^-1]
λ_3	0.89 [cm^-1*Torr^-1]
p_{O2}	150 [Torr]
f_q	0.037975

Material properties as a function of reduced electric field E_{Td} – DDR equations

Tab. 3: Transport coefficients for positive ions p [5].

μ_p	2.0e-4 [m^2/V/s]	Positive ion p mobility
D_p	5.05e-6 [m^2/s]	Positive ion p diffusivity

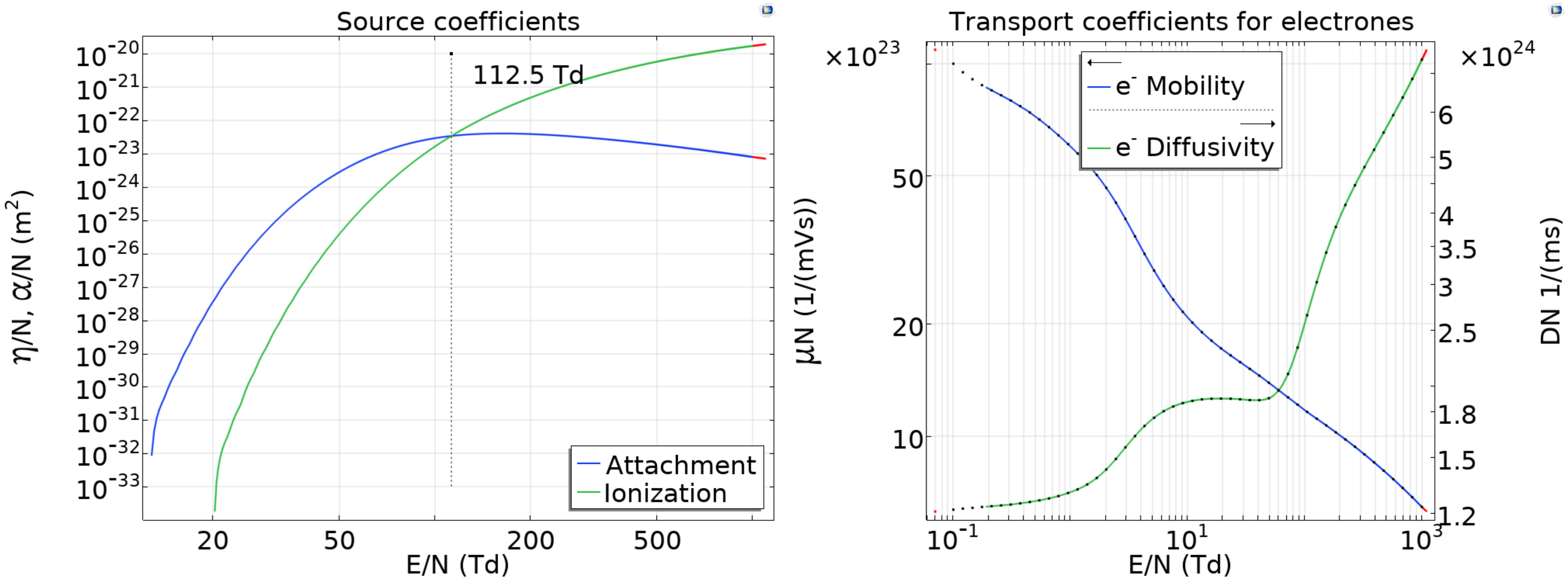


Fig. 5: Source (ionization α and attachment η) and transport (electron e mobility μ_e and diffusivity D_e) coefficients as a functions of reduced electric field E/N (Td) [6].

Material properties as a function of reduced electric field E_{Td} – Photoionization

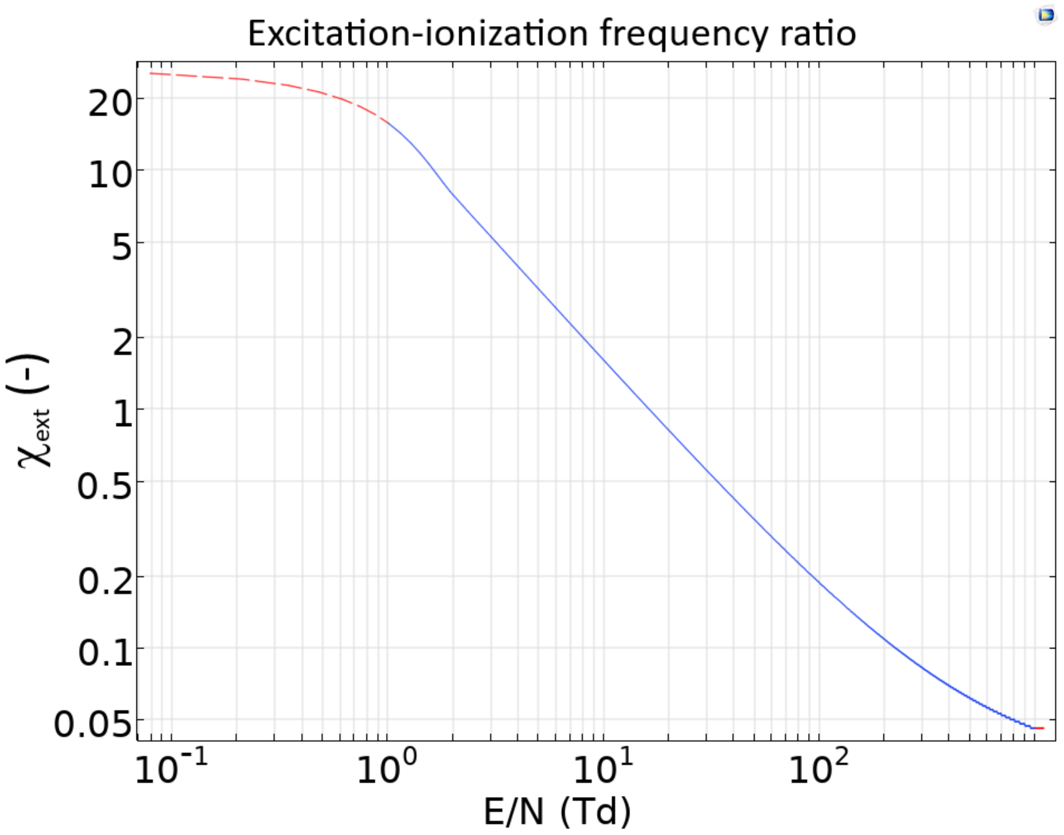
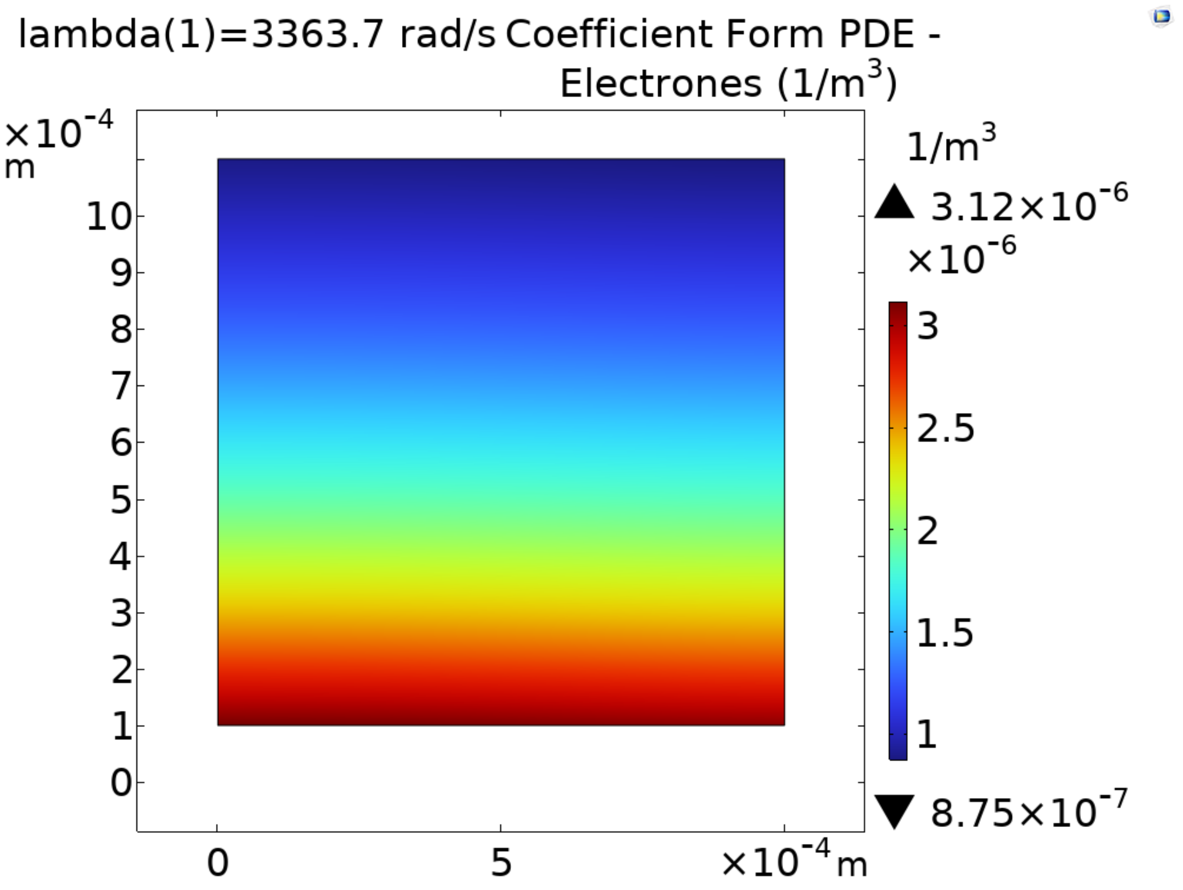


Fig. 6: Excitation-ionization frequency ratio χ_{ext} from slide 7 as a functions of reduced electric field *E*/*N* (Td) for photoionization model [4].

U → λ study – Plane



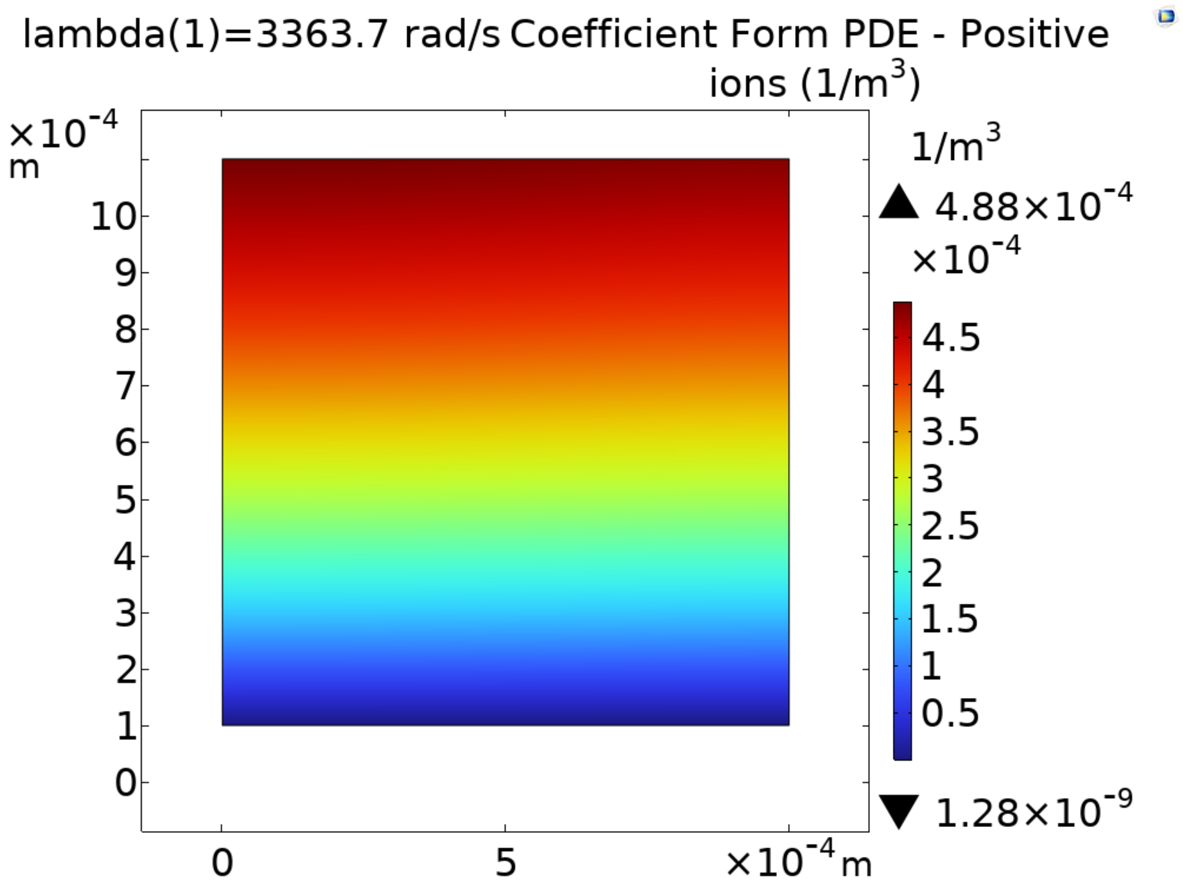


Fig. 7: Electron and positive ion concentrations for plane – $\lambda_c = U_c = 3363.7$ V.

U → λ study – Protrusion no. 1

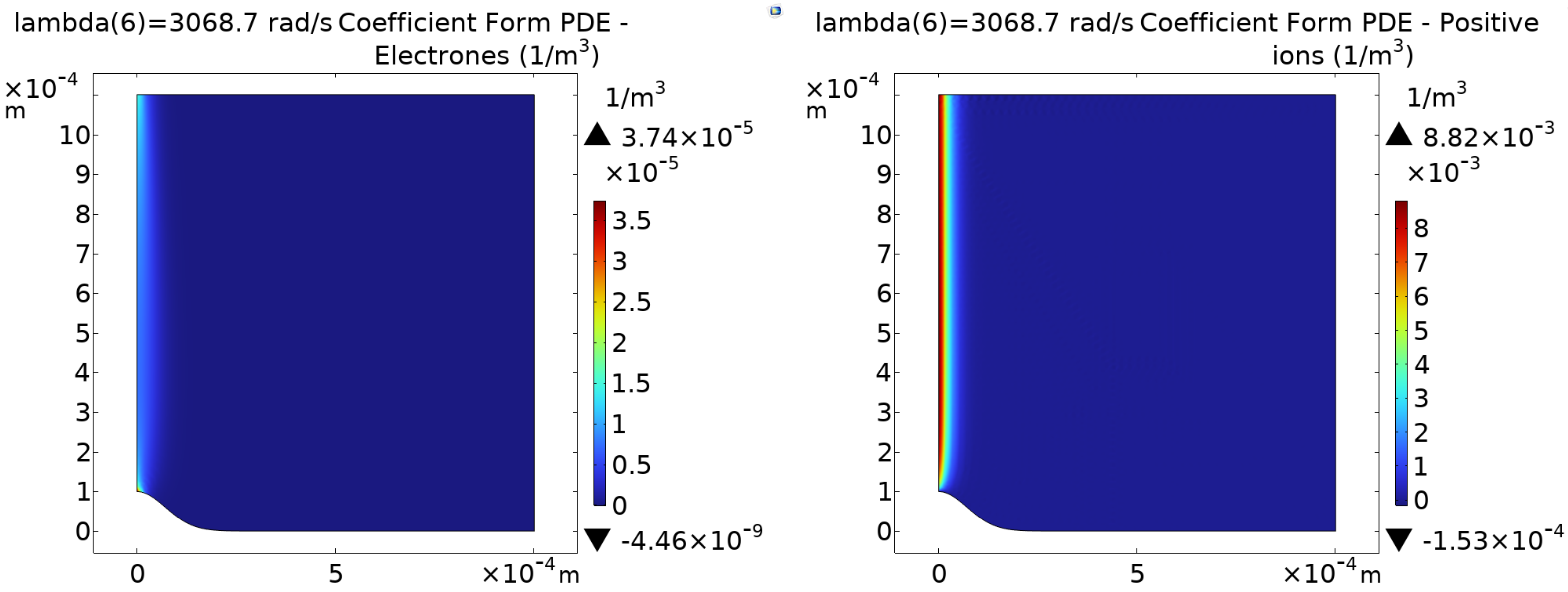


Fig. 8: Electron and positive ion concentrations for protrusion no. 1 – $\lambda_c = U_c$ = 3068.7 V.

U → λ study – Protrusion no. 2

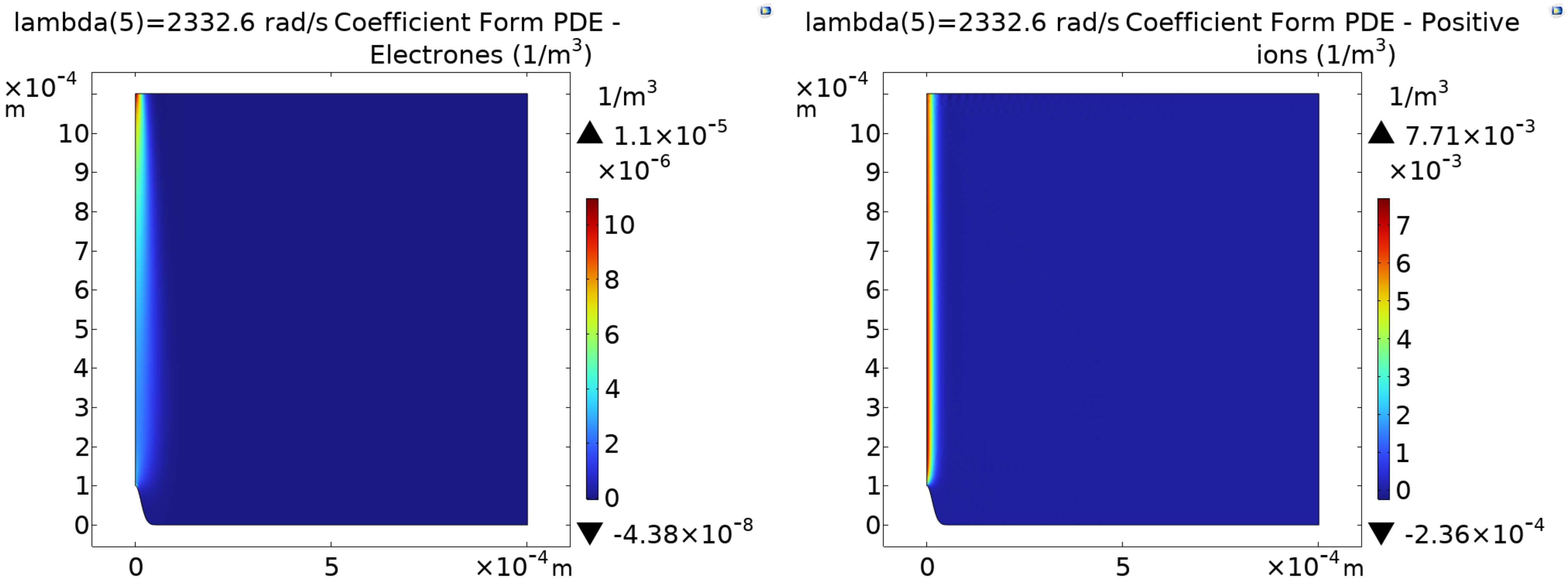


Fig. 9: Electron and positive ion concentrations for protrusion no. 2 – $\lambda_c = U_c$ = 2332.6 V.

Townsend criteria[2]

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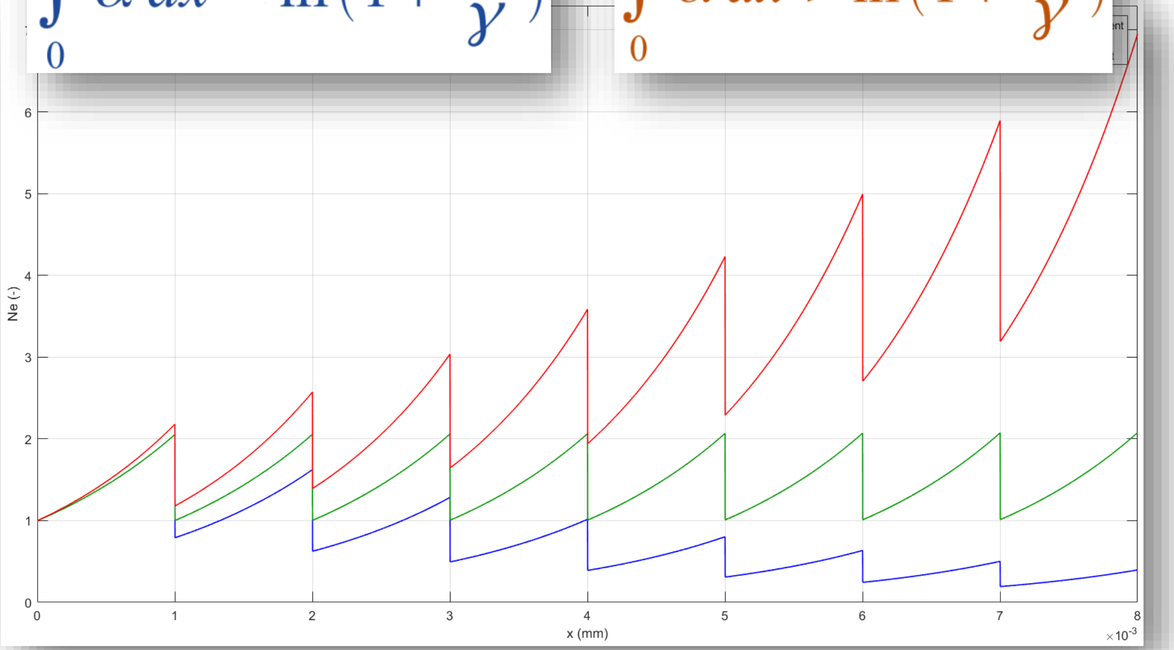

Fig. 10: Non-self-sustaining (blue) and Self-sustaining (red) discharge, critical state - green.

Townsend criteria – 1D integration

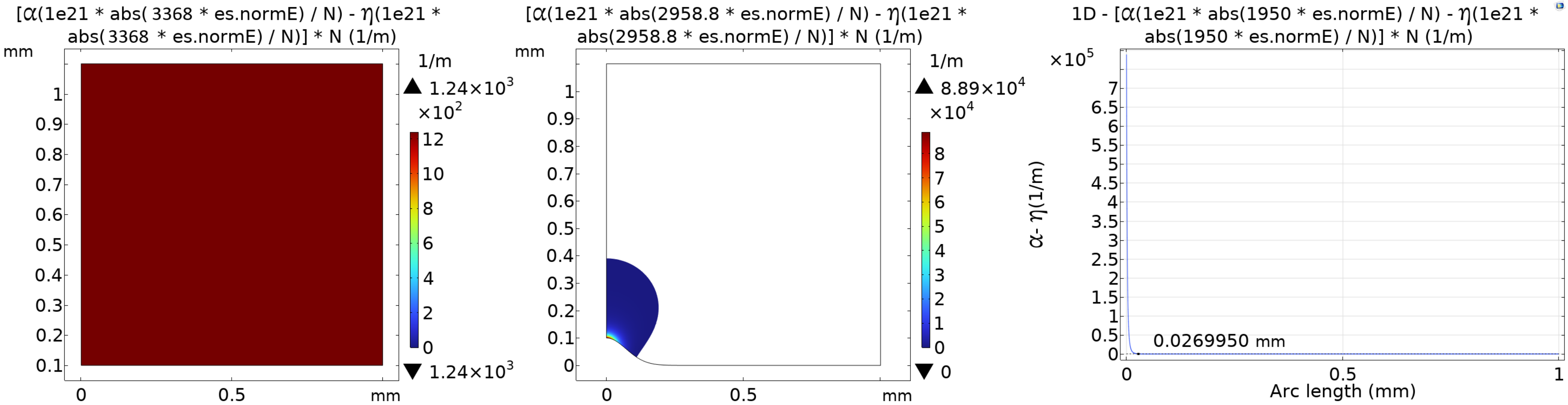

Fig. 11: Positive part of α - η for plane (U_c = 3368 V), protrusion no. 1 (U_c = 2958.8 V) and for protrusion no. 2 only 1D section along z-axis (U_c = 1950 V).

- Quantity E_{Td} in $\alpha(E_{Td})$, $\eta(E_{Td})$, and others exceeded beyond available date shown in Fig. 5 and Fig. 6 for protrusion no. 2 – this case will not provide correct result

Comparison & Conclusions

- **U → λ study** and **Townsend criteria** for calculating self-sustaining voltage U_c – transition between non-self-sustaining and self-sustaining discharge:

1. **U** → λ study of non-trivial solutions
   - Eigenvalue study in terms of voltage *U* analyses equilibrium between gain and loss of charged particles – self-sustaining discharge
2. **Townsend criteria** – 1D integration of $\alpha - \eta$ along z-axis

$$\int_0^d \alpha\, dx < \ln\left(1+\frac{1}{\gamma}\right) \qquad \int_0^d \alpha\, dx \geq \ln\left(1+\frac{1}{\gamma}\right)$$

- Values of U_c for **protrusion no. 2** are different because quantity E_{Td} in $\alpha(E_{Td})$, $\eta(E_{Td})$, and others exceeded beyond available date

Tab. 4: Comparison of the calculated self-sustaining voltages U_c by eigenvalue study and Townsend criteria.

d = 1 mm	***U*** **→ λ**	**∫(α - η) dz**
Plane	3363.7	3368.0
Protrusion no. 1	3068.7	2958.8
Protrusion no. 2	2332.6	1950.0

Thank you for your attention

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[1] COMSOL Multiphysics® v. 6.1. www.comsol.com. COMSOL AB, Stockholm, Sweden.
[2] TOWNSEND, John. The theory of ionization of gases by collision. Constable, Limited, 1910.
[3] BENILOV, M. S., et al. A practical guide to modeling low-current quasi-stationary gas discharges: Eigenvalue, stationary, and time-dependent solvers. Journal of Applied Physics, 2021, 130.12.
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[5] SERDYUK, Yuriy V. Propagation of cathode-directed streamer discharges in air. In: Proc. COMSOL Conf., Rotterdam, The Netherlands. 2013.
[6] G.J.M. Hagelaar and L.C. Pitchford, "Solving the Boltzmann equation to obtain electron transport coefficients and rate coefficients for fluid models", Plasma SciSources and Tech 14, 722 (2005).

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