

# Curvilinear Coordinates

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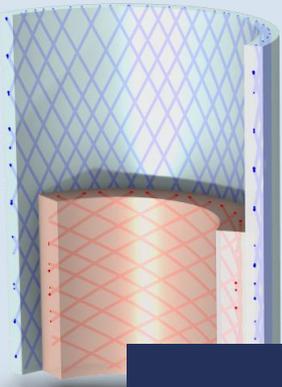
Development Manager,  
COMSOL

# Coordinate Systems

How to set up and use different coordinate systems in COMSOL.

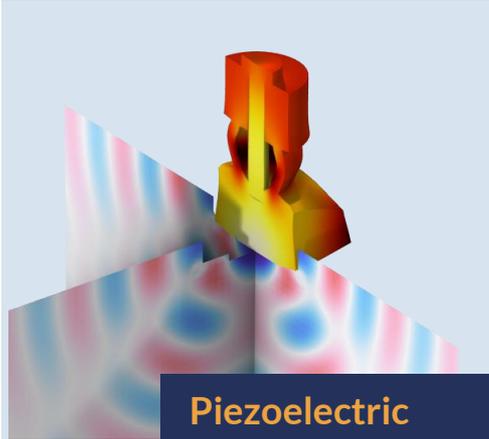


# Application Areas



## Biological Tissue

Stiffness of muscle tissue (fibers) or arterial walls depend on direction.



## Piezoelectric Material

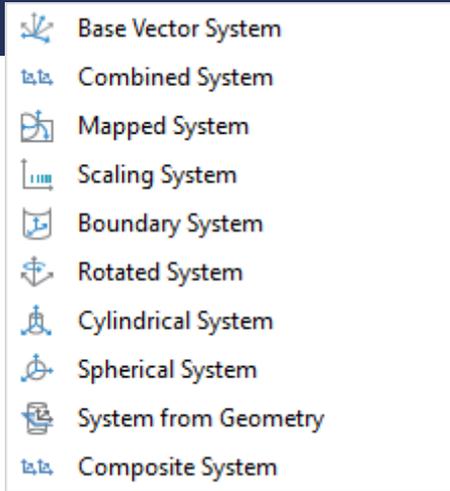
Materials have direction dependent properties. Align coordinate system with crystal axes or polarization direction.



## Composites

Composites often have anisotropic properties (e.g. fibers). Especially in layered structures these often vary for each layer.

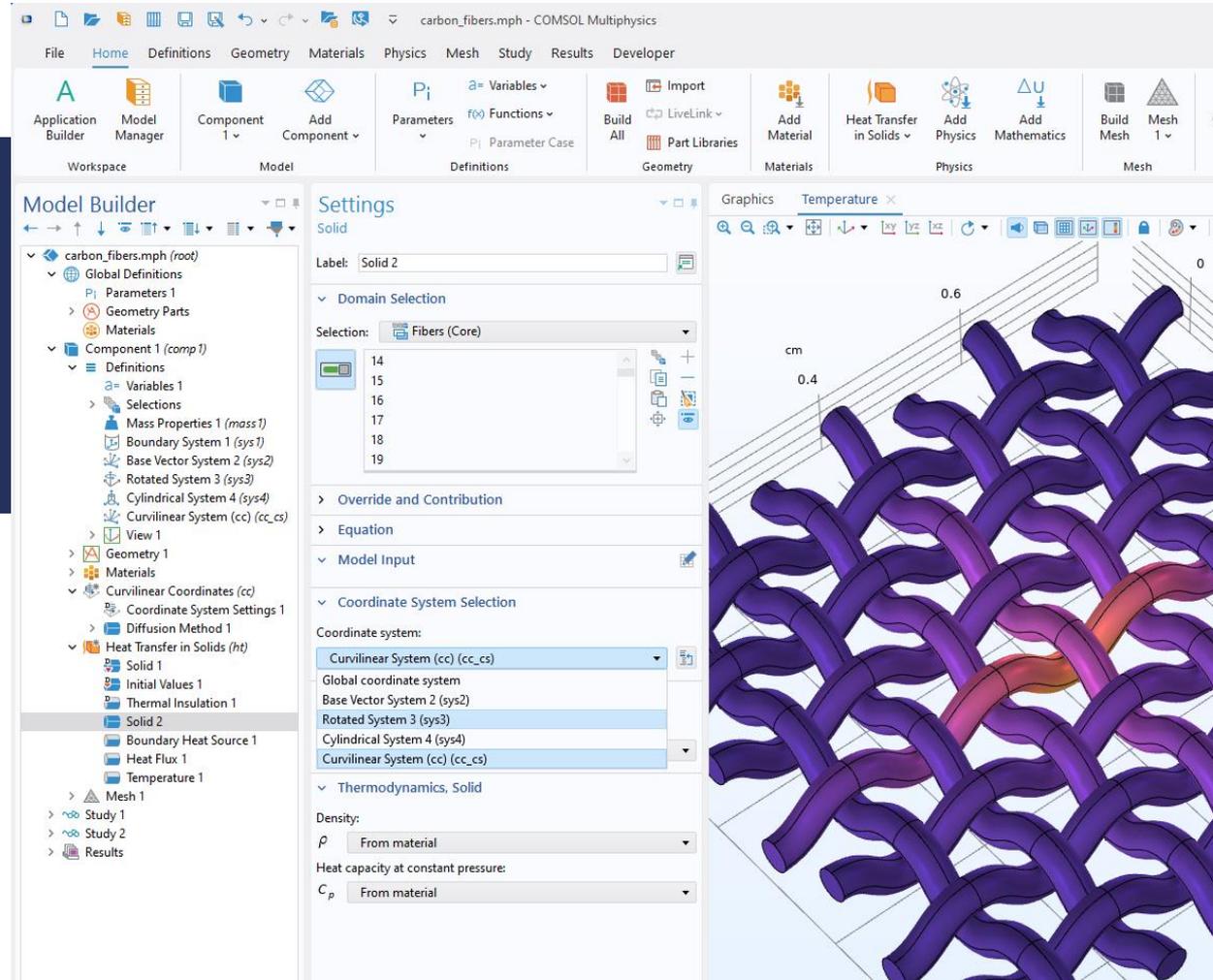
# Coordinate Systems in COMSOL



- Global cartesian coordinate system shown in Graphics window
- User-defined coordinate systems can be used to define material properties or for physics settings.
- COMSOL uses global system to solve the equations
- Coordinate system transformations are done automatically

# Use Coordinate Systems in Physics Features

- The selected coordinate systems will be used for all tensor or vector inputs
- COMSOL automatically computes the transformation to the global system



## Example: Heat Transfer

- Input of user-defined coordinate systems is only available in features that can make use of it
- Use the coordinate system to define the thermal properties
- Tensor-valued properties automatically use the selected coordinate system
- Scalar values are not affected

The screenshot shows the COMSOL Model Builder interface. The left pane displays the 'Model Builder' tree for a model named 'carbon\_fibers.mph'. The tree is expanded to show 'Solid 2' under 'Heat Transfer in Solids (ht)'. The right pane shows the 'Settings' for 'Solid 2'. The 'Coordinate System Selection' section is expanded, showing 'Coordinate system:' set to 'Curvilinear System (cc) (cc\_cs)'. Below this, the 'Heat Conduction, Solid' section shows 'Thermal conductivity:' set to 'k' with a dropdown menu set to 'From material'. The 'Thermodynamics, Solid' section shows 'Density:' set to  $\rho$  with a dropdown menu set to 'From material', and 'Heat capacity at constant pressure:' set to  $C_p$  with a dropdown menu set to 'From material'.

Coordinate System

Tensor

Scalar

## Example: Heat Transfer

- Which equations and variables are computed can be inspected in the equation view
- Variables are transformed automatically into global system
- Equations and other variables are not transformed

Settings  
Equation View

Label: Equation View

> Study

Variables

Name	Expression
ht.lxz	(spatial.F13*(cc_cs.T11*(spatial.F11*(cc_cs.T11*ht.solid2.k_locale1e1+cc_cs.T21*...))
ht.kyx	(spatial.F11*(cc_cs.T11*(spatial.F12*(cc_cs.T11*ht.solid2.k_locale1e1+cc_cs.T21*...))
ht.kyy	(spatial.F12*(cc_cs.T11*(spatial.F12*(cc_cs.T11*ht.solid2.k_locale1e1+cc_cs.T21*...))
ht.kvz	(spatial.F13*(cc_cs.T11*(spatial.F12*(cc_cs.T11*ht.solid2.k_locale1e1+cc_cs.T21*...))

Shape Functions

Weak Expressions

Weak expression	Integration order	Frame
(ht.dfluxx*test(Tx)+ht.dfluxy*test(Ty)+ht.dfluxz*test(Tz))*ht.d	4	Spatial

Computation of thermal conductivity tensor components

Equation to solve

# Thermal Conductivity

- Define components of thermal conductivity (isotropic, diagonal, symmetric, full)
- Which coordinate system is used is determined by the physics feature.

The screenshot shows the COMSOL Model Builder interface. The left pane displays the Model Builder tree with the following structure:

- carbon\_fibers.mph (root)
  - Global Definitions
    - Parameters 1
    - Geometry Parts
    - Materials
  - Component 1 (comp 1)
    - Definitions
    - Geometry 1
    - Materials
      - Epoxy (mat1)
      - Carbon (mat2)
      - Carbon (Infinite Element Domain)
    - Curvilinear Coordinates (cc)
      - Coordinate System Settings 1
      - Diffusion Method 1
    - Heat Transfer in Solids (ht)
      - Solid 1
        - Initial Values 1
        - Thermal Insulation 1
        - Solid 2
        - Boundary Heat Source 1
        - Heat Flux 1
        - Temperature 1
    - Mesh 1
    - Study 1
    - Study 2

The right pane shows the Settings window for the selected material, Carbon (mat2). The Geometric Entity Selection is set to Domain, and the Selection is Fibers. The Material Contents table is shown below:

Property	Variable	Value	Unit
<input checked="" type="checkbox"/> Thermal conductivity	{k11, k22, ...}	{60, 4, 4}	W/(m·K)
<input checked="" type="checkbox"/> Density			
<input checked="" type="checkbox"/> Heat capacity			

A dialog box titled "Thermal conductivity" is open, showing a Diagonal matrix configuration:

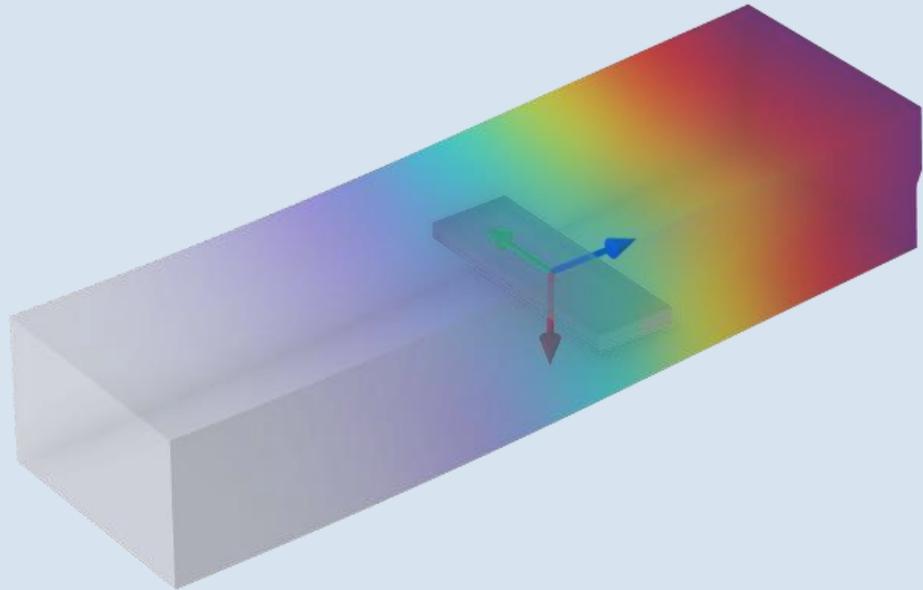
Diagonal		
60	0	0
0	4	0
0	0	4

Buttons for OK and Cancel are visible at the bottom of the dialog.

DEMO

# Piezoelectric Actuator

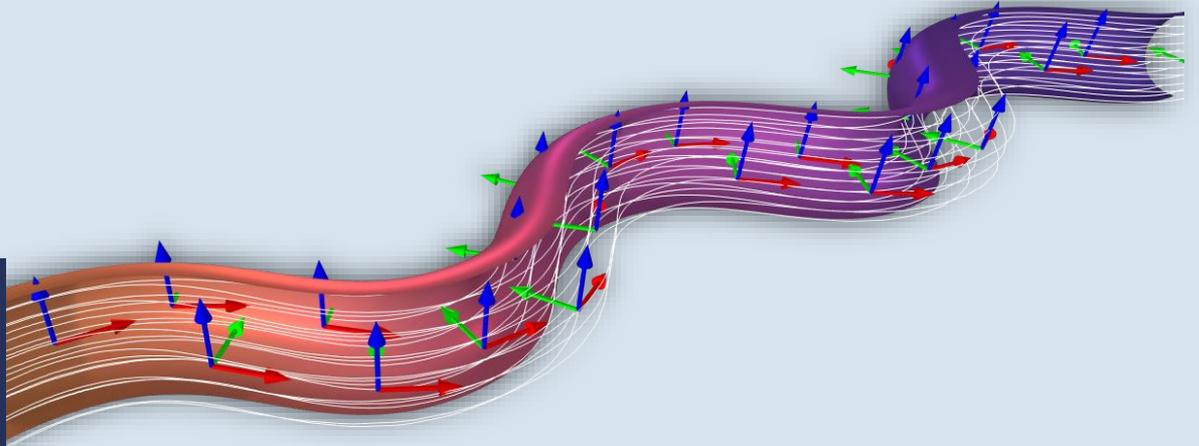
How to use and set up different coordinate systems.



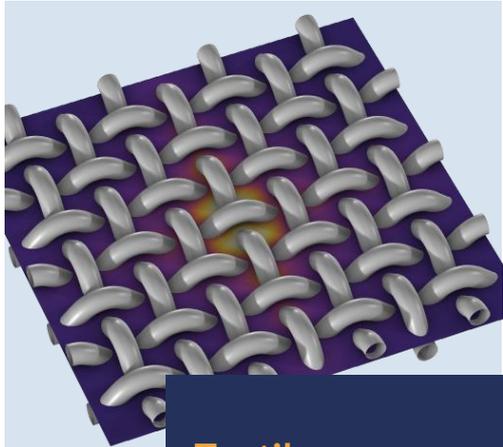
FOLLOW SHAPE

# Curvilinear Coordinates

Compute a coordinate system if a basic coordinate system is too difficult to use, e.g. for fibrous material in complex shapes

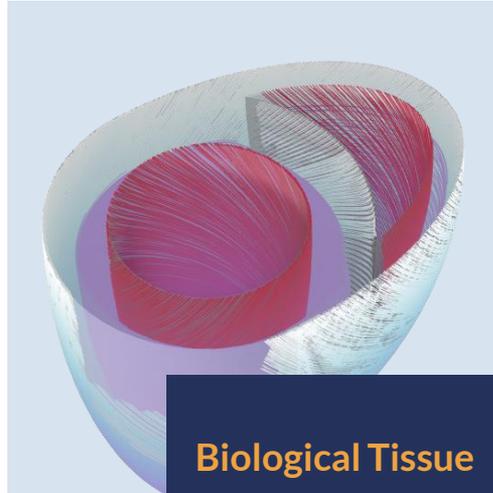


# Application Areas



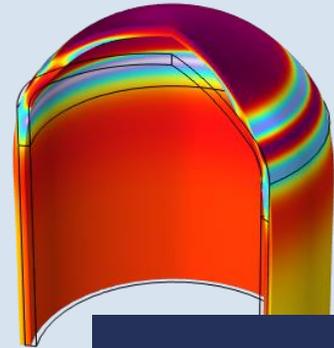
## Textiles

Woven fabric, where properties are aligned with the fiber direction.



## Biological Tissue

Muscle tissue around heart has anisotropic properties aligning with fiber orientation.



## Composites

Rolled steel sheet has different mechanical directions in out-of-plane direction

First basis vector is defined by the solution of the equation (gradient).

Settings

Second basis vector:

x-axis

Selected Method

The screenshot shows the COMSOL Model Builder interface. The left pane shows a tree view with 'Curvilinear Coordinates (cc)' selected. The right pane shows the 'Settings' for 'Curvilinear Coordinates'. The 'Domain Selection' is set to 'Manual'. A list of domains (14-19) is visible. Under the 'Settings' section, 'Normalize vector field' and 'Create base vector system' are checked. The 'Equation' and 'Discretization' sections are partially visible.

Normalize vector field (default) to prevent scaling effects

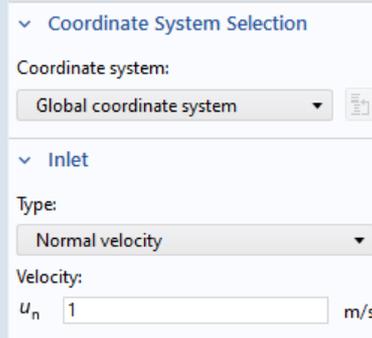
To define a coordinate system under **Definitions**

## Inlet

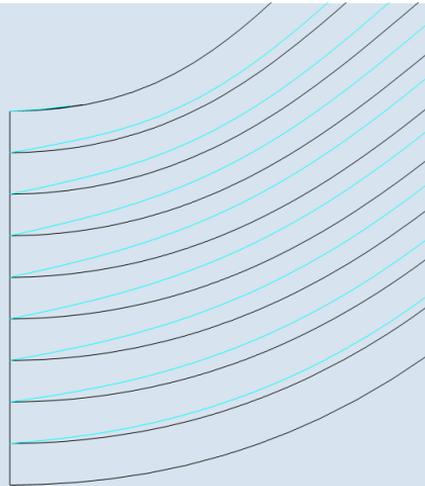
- Defines starting of the vector field
- Settings differ for different methods.

## Outlet

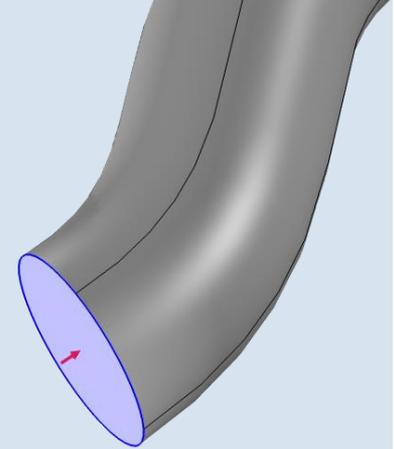
- Defines end of vector field



*Flow method allows using other coordinate systems to define direction.*

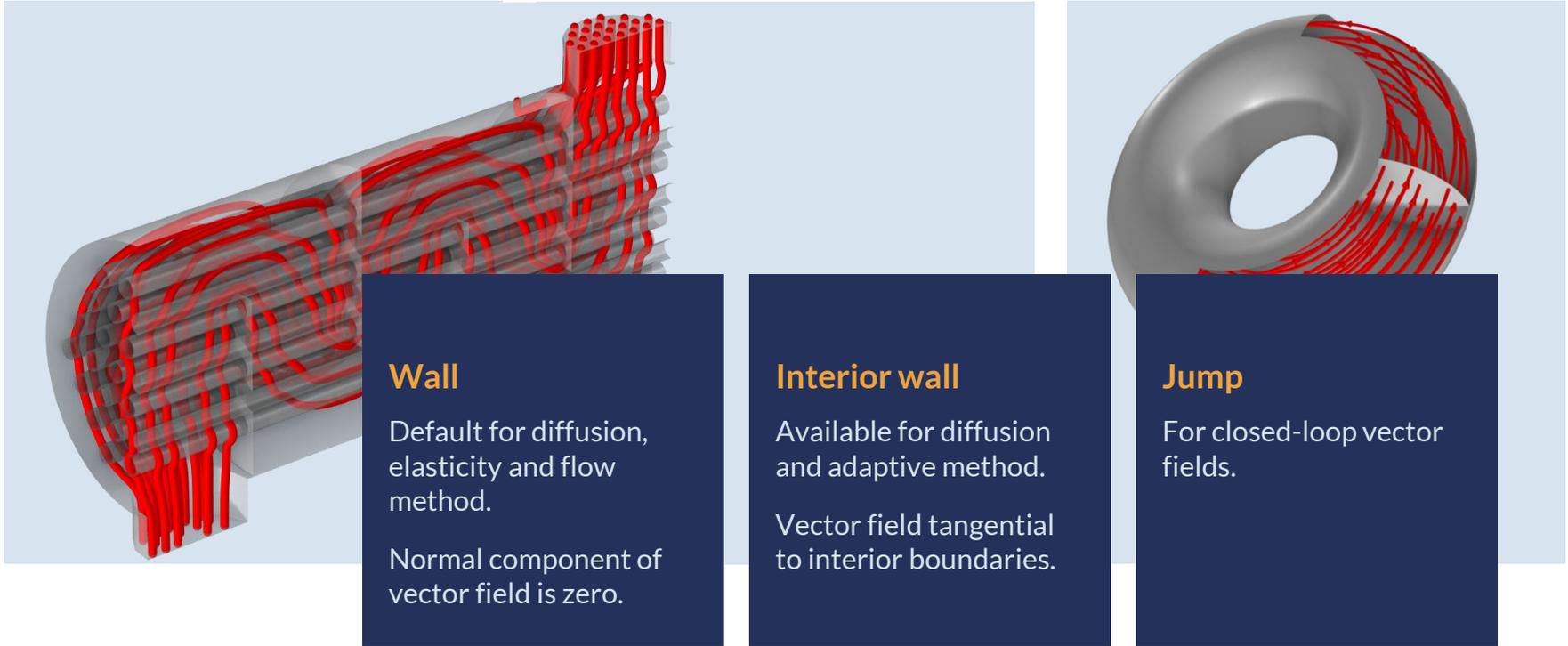


*Flux (blue) or constraint (black) inlet for diffusion method*



*Vector field enters orthogonally in the adaptive method.*

# Wall, Interior Wall, Jump



## Wall

Default for diffusion, elasticity and flow method.

Normal component of vector field is zero.

## Interior wall

Available for diffusion and adaptive method.

Vector field tangential to interior boundaries.

## Jump

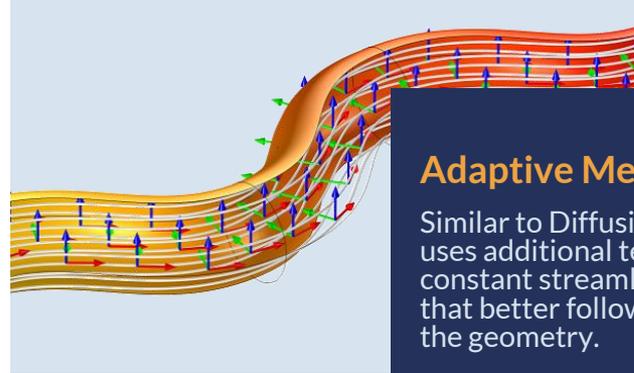
For closed-loop vector fields.

# Methods



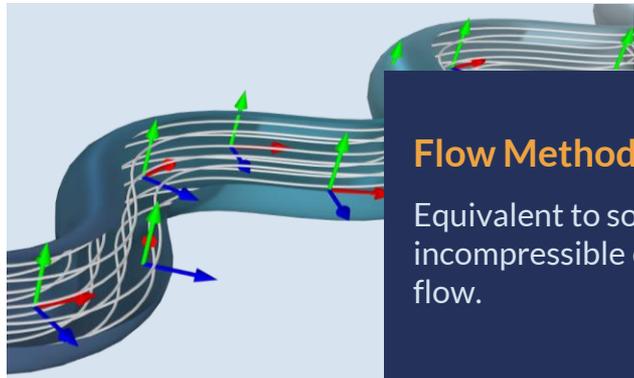
## Diffusion Method

Solution of Laplace's equation (heat conduction, diffusion). Vector field points in direction of steepest gradient.



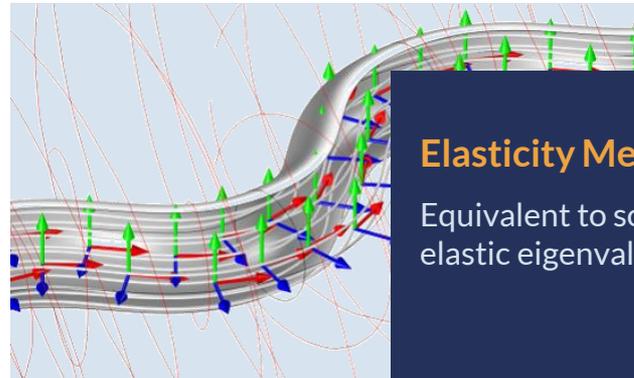
## Adaptive Method

Similar to Diffusion method but uses additional terms for more constant streamline density that better follows the shape of the geometry.



## Flow Method

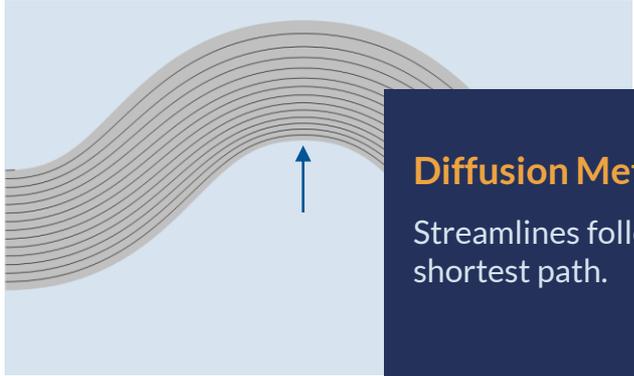
Equivalent to solving incompressible creeping flow.



## Elasticity Method

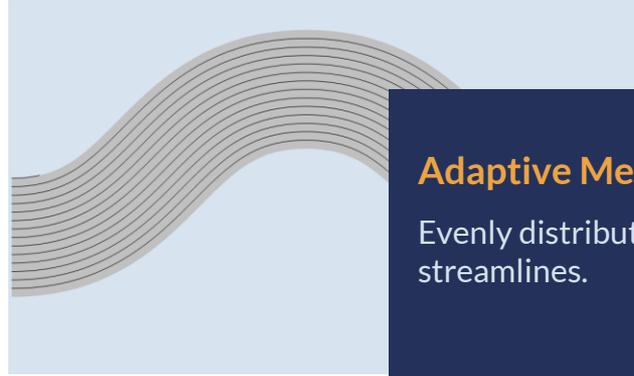
Equivalent to solving a linear elastic eigenvalue problem.

# Curvature



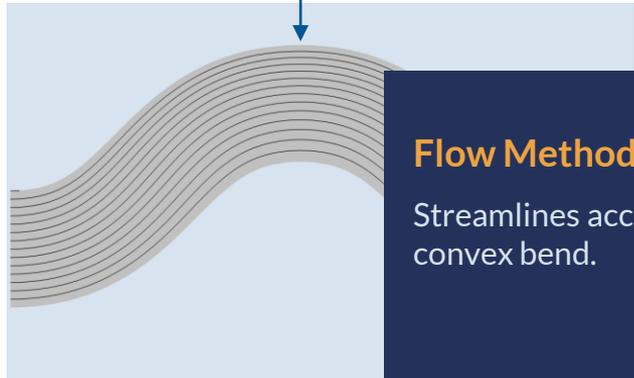
## Diffusion Method

Streamlines follow the shortest path.



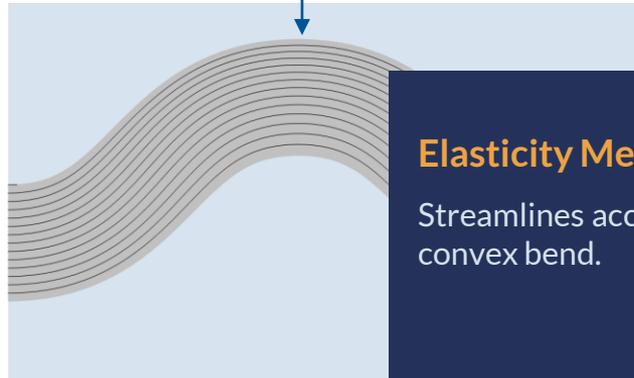
## Adaptive Method

Evenly distributed streamlines.



## Flow Method

Streamlines accumulate at convex bend.



## Elasticity Method

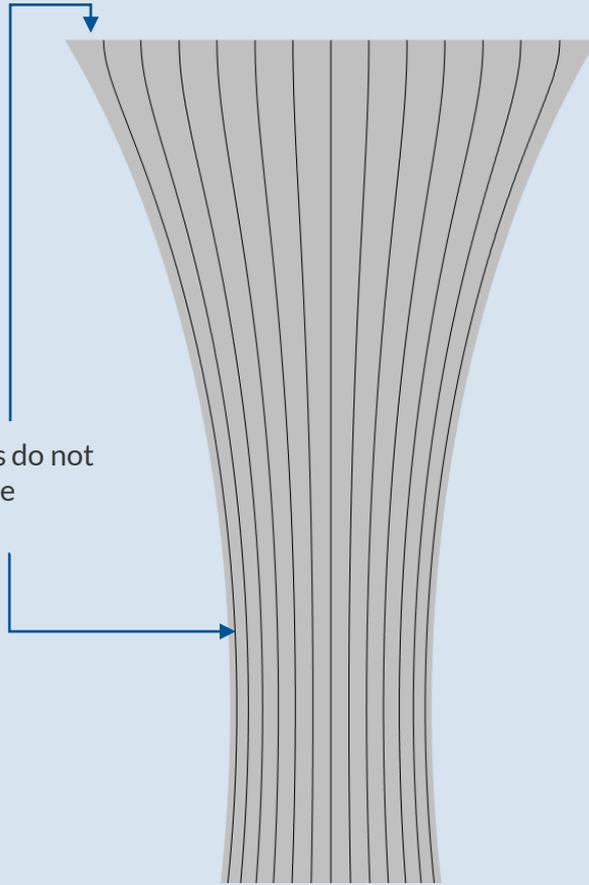
Streamlines accumulate at convex bend.

# Variable Cross Section

Diffusion Method

Streamlines do not follow shape perfectly.

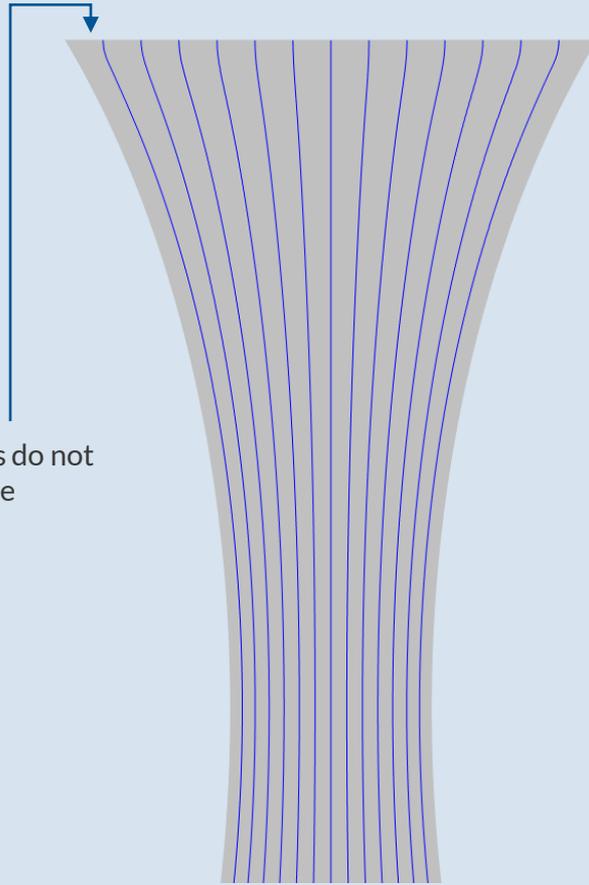
- Diffusion
- Adaptive
- Flow
- Elasticity



# Variable Cross Section

## Adaptive Method

Streamlines do not follow shape perfectly.



■ Diffusion

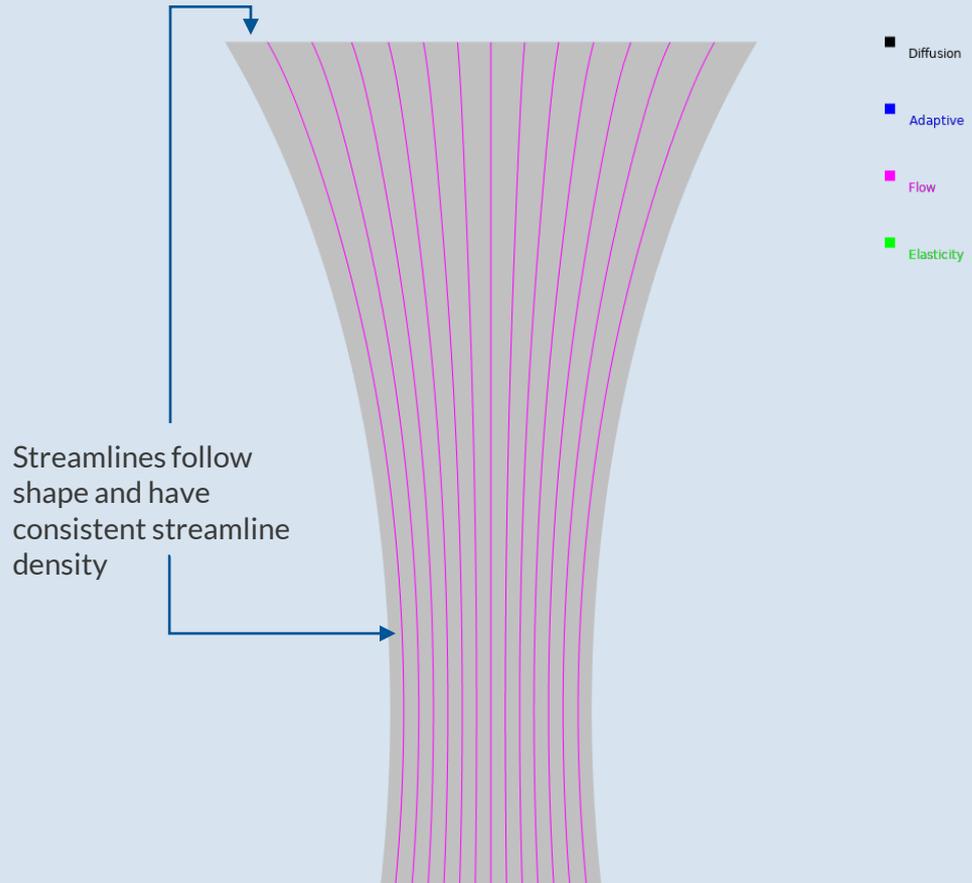
■ Adaptive

■ Flow

■ Elasticity

# Variable Cross Section

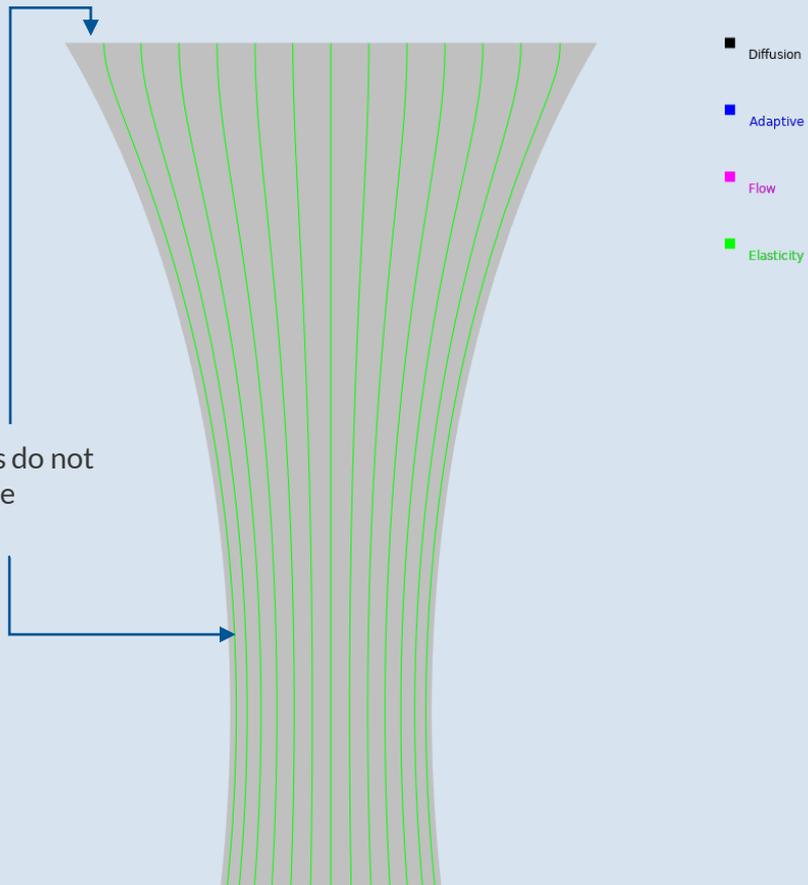
## Flow Method



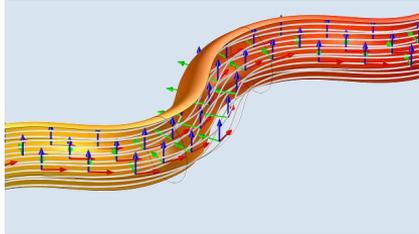
# Variable Cross Section

Elasticity Method

Streamlines do not follow shape perfectly.

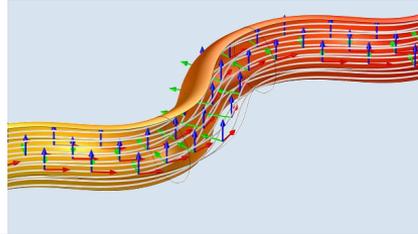


# Summary



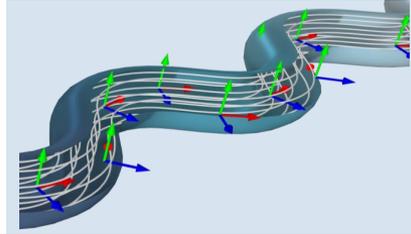
## Diffusion

- + Simple and fast
- Not ideal for sharp bends and corners



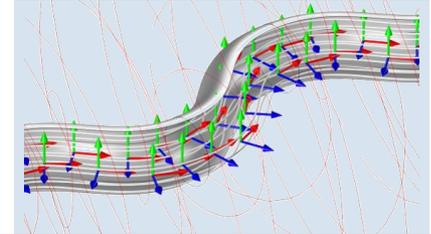
## Adaptive

- + Simple and fast
- Not ideal for variable cross sections



## Flow

- + Robust
- + Can handle most geometries
- High computational cost



## Elasticity

- Good geometry handling
- Not easy to solve for all cases, manual searching for eigenvalues

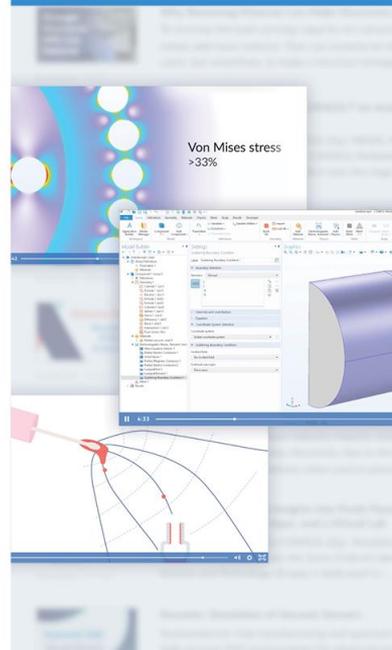
# Further Resources for Inspiration

comsol.com

## BLOG POSTS



## VIDEOS



## USER STORIES



## MODELS & APPLICATIONS

