Uncertainty Quantification Module

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  - Mixer Module
- Polymer Flow Module
- Microfluidics Module
- Porous Media Flow Module
- Subsurface Flow Module
- Pipe Flow Module
- Molecular Flow Module
- Metal Processing Module
- Heat Transfer Module

STRUCTURAL & ACOUSTICS

- Structural Mechanics Module
  - Nonlinear Structural Materials Module
  - Composite Materials Module
  - Geomechanics Module
  - Fatigue Module
  - Rotordynamics Module
- Multibody Dynamics Module
- MEMS Module
- Acoustics Module

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- Chemical Reaction Engineering Module
- Battery Design Module
- Fuel Cell & Electrolyzer Module
- Electrodeposition Module
- Corrosion Module
- Electrochemistry Module

MULTIPURPOSE

- Optimization Module
- Uncertainty Quantification Module
- Material Library
- Particle Tracing Module
- Liquid & Gas Properties Module

50. Module !!

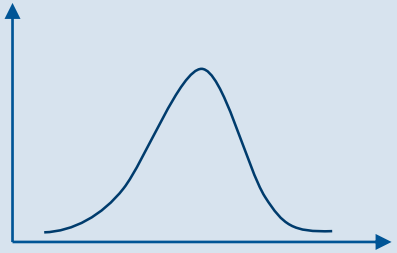
INTERFACING

- LiveLink™ *for* MATLAB®
- LiveLink™ *for* Simulink®
- LiveLink™ *for* Excel®
- CAD Import Module
- Design Module
- ECAD Import Module
- LiveLink™ *for* SOLIDWORKS®
- LiveLink™ *for* Inventor®
- LiveLink™ *for* AutoCAD®
- LiveLink™ *for* Revit®
- LiveLink™ *for* PTC® Creo® Parametric™
- LiveLink™ *for* PTC® Pro/ENGINEER®
- LiveLink™ *for* Solid Edge®
- File Import *for* CATIA® V5

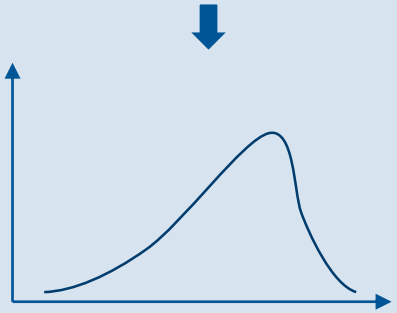
Uncertainty Quantification Module

- Current methodology:
  - Deterministic design with safety margins/tolerances and worst-case-scenario analyses
- Uncertainty quantification (UQ):
  - Probabilistic design
  - Quantifies „How likely is it to fail?“
- Based on statistical methods
- In COMSOL Multiphysics
  - UQ for any physics!

What is the probability that my design will meet set design criteria considering uncertainties in manufacturing?



Input Distribution

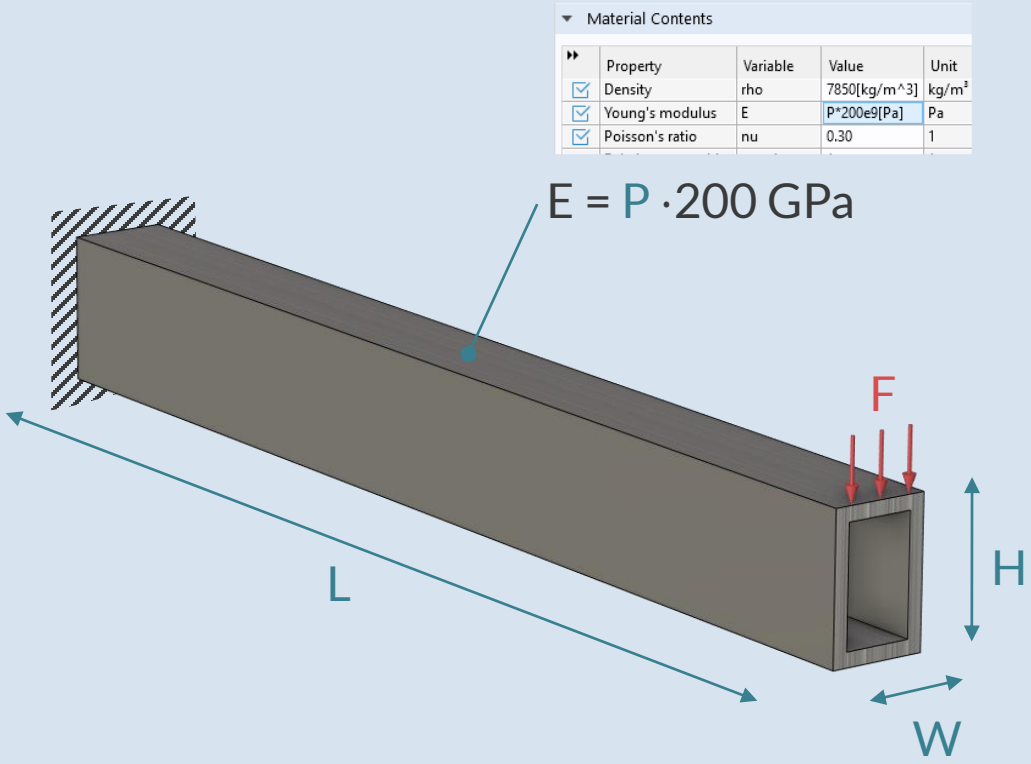


Output Distribution

The Uncertainty Quantification Module

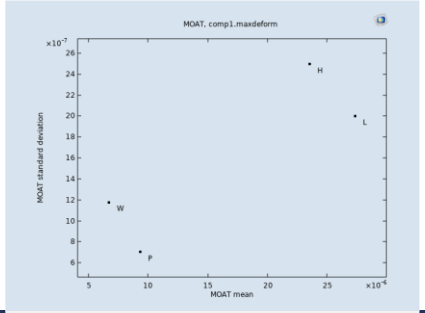
How the quantities of interest depend on variations in the inputs of a model?

- Input Parameters
  - Design parameters
  - Material parameters
  - Mesh parameters
  - Physics settings parameters
- Quantities of Interests (QoI)
  - Max displacement
  - Max temperature
  - Resistance, Capacitance…



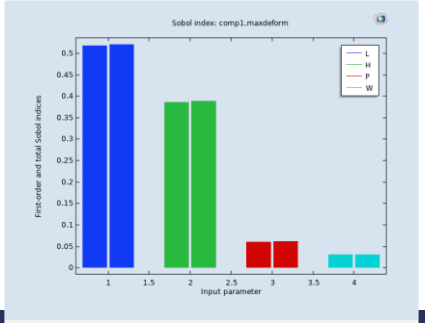
My Uncertainty Quantification example:
Input Parameters: W, H, L, P
Variation of inputs: ± 5%
Quantities of Interests: maximal displacement

Uncertainty Quantification Module Study Types



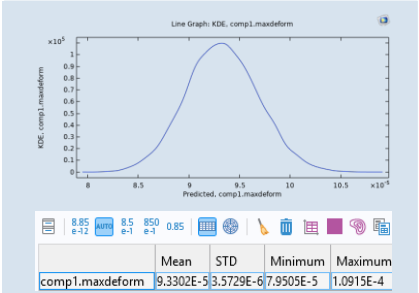
Screening

Identify the most important input parameters



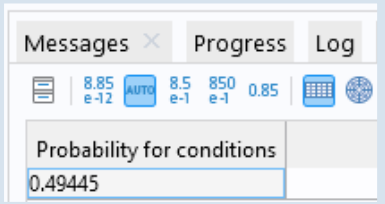
Sensitivity Analysis

Determine output sensitivity with respect to the input parameters



Uncertainty Propagation

Determine output probability distribution given the input probability distributions



Reliability analysis

Determine the probability that outputs satisfy reliability criteria

New

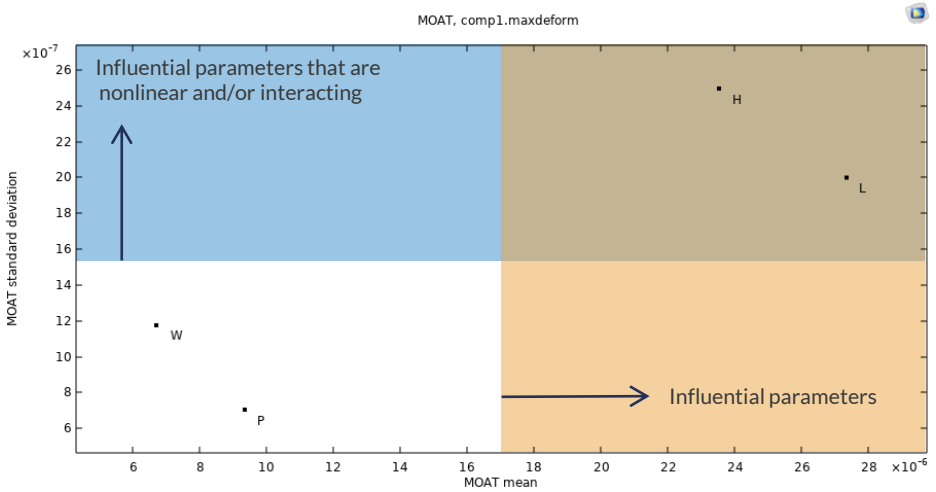
Model Wizard

Blank Model

Help Cancel Show on startup

Uncertainty Quantification Module Study Types

- Screening:
  - Identify the most important input parameters
  - Sample based Morris one-at-a-time (MOAT) method
  - Ideal when the number of input parameters is too large
  - High MOAT mean: parameter is significantly influencint the QoI
  - High MOAT standard deviation: parameter strongly interacts with other parameters or it has nonlinear influence (or both)



The *Screening, MOAT* study type implements a lightweight global screening method that gives a qualitative measure of the importance of each input parameter.

Untitled.mph - COMSOL Multiphysics

File Home Definitions Geometry Materials Physics Mesh Study Results Developer

Application Builder | Model Manager | Component 1 | Add Component | Parameters | Variables | Functions | Parameter Case | Build All | Import | LiveLink | Add Material | Solid Mechanics | Add Physics | Build Mesh | Mesh 1 | Compute | Study 1 | Add Study | Stress (solid) | Add Plot Group | Windows | Reset Desktop

Workspace | Model | Definitions | Geometry | Materials | Physics | Mesh | Study | Results | Layout

Model Builder

- Untitled.mph *(root)*
  - Global Definitions
    - Parameters 1
    - Materials
  - Component 1 *(comp 1)*
    - Definitions
      - Variables 1
      - Maximum 1 *(maxop 1)*
      - Boundary System 1 *(sys1)*
      - View 1
    - Geometry 1
      - Work Plane 1 *(wp 1)*
        - Plane Geometry
          - Rectangle 1 *(r1)*
          - Offset 1 *(off1)*
          - Difference 1 *(dif1)*
      - View 2
      - Extrude 1 *(ext1)*
      - Form Union *(fin)*
    - Materials
      - Structural steel *(mat1)*
    - Solid Mechanics *(solid)*
      - Linear Elastic Material 1
      - Free 1
      - Initial Values 1
      - Edge Load 1
      - Fixed Constraint 1
    - Mesh 1
  - Study 1
    - Step 1: Stationary
    - Solver Configurations
  - Results
    - Datasets
    - Derived Values
      - Global Evaluation 1
    - Tables
    - Stress (solid)
    - Applied Loads (solid)
      - Edge Loads (solid)
    - Export
    - Reports

Settings

Global Evaluation

Evaluate

Label: Global Evaluation 1

Data

Dataset: Study 1/Solution 1 (sol1)

Expressions

Expression	Unit	Description
maxdeform	mm	

Expression:

Description:

Parameters

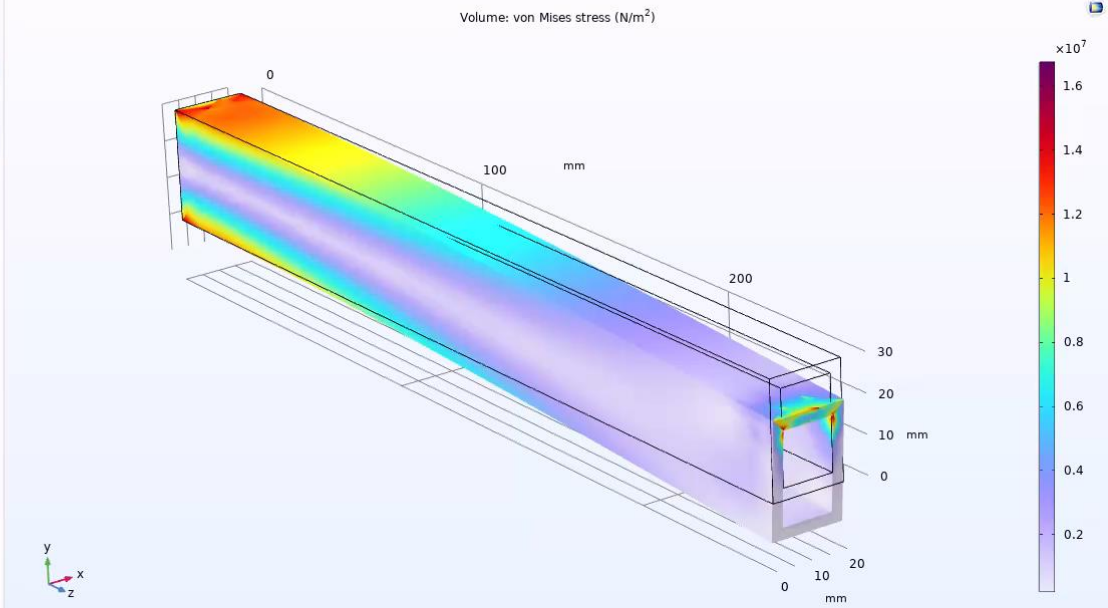
Name	Value	Unit	Description
solid.refpntx	0	m	Reference point for moment computation, x coordi...
solid.refpnty	0	m	Reference point for moment computation, y coordi...
solid.refpntz	0	m	Reference point for moment computation, z coordi...

Data Series Operation

Normalization: None

Transformation: None

Graphics



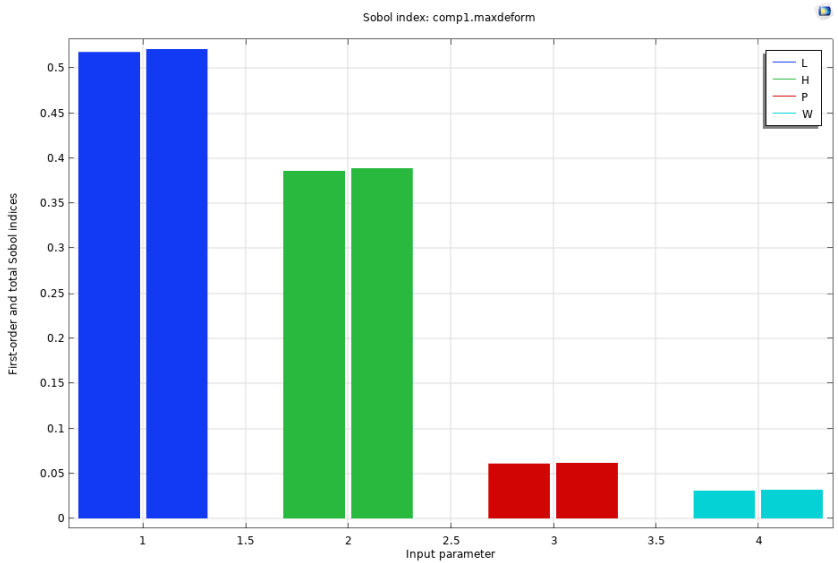
Messages | Progress | Log | Table 1

maxdeform (mm)
0.093236

1.46 GB | 1.78 GB

Uncertainty Quantification Module Study Types

- Screening
- Sensitivity Analysis:
  - Determine output sensitivity with respect to the input parameters
  - Two methods implemented
    - Sobol method
    - Correlation method
  - The quantity of interest is most sensitive to the input parameter with the highest total Sobol index.



The Sobol method analyzes the entire input-parameter distribution and decomposes the variance of each quantity of interest into a sum of contributions from the input parameters and their interactions.

Untitled.mph - COMSOL Multiphysics

File Home Definitions Geometry Materials Physics Mesh Study Results Developer MOAT, comp1.maxdeform

Plot Plot In Line Graph Point Graph Global Table Graph Line Segments Histogram Mesh Annotation More Plots Color Expression Filter Graph Marker More Attributes Image Animation

Plot Add Plot Attributes Export

Model Builder

- Untitled.mph *(root)*
  - Global Definitions
    - Parameters 1
    - Materials
  - Component 1 *(comp1)*
    - Definitions
      - Variables 1
      - Maximum 1 *(maxop1)*
      - Boundary System 1 *(sys1)*
      - View 1
    - Geometry 1
      - Work Plane 1 *(wp1)*
        - Plane Geometry
          - Rectangle 1 *(r1)*
          - Offset 1 *(off1)*
          - Difference 1 *(dif1)*
        - View 2
      - Extrude 1 *(ext1)*
      - Form Union *(fin)*
    - Materials
      - Structural steel *(mat1)*
    - Solid Mechanics *(solid)*
      - Linear Elastic Material 1
      - Free 1
      - Initial Values 1
      - Edge Load 1
      - Fixed Constraint 1
    - Mesh 1
  - Study 1
  - Study 2 - Screening MOAT
  - Results
    - Uncertainty Quantification Graph
      - MOAT, comp1.maxdeform
    - Datasets
    - Derived Values
      - Global Evaluation 1
    - Tables
    - Stress (solid)
    - Applied Loads (solid)
      - Edge Loads (solid)
    - Export
    - Reports

Settings

1D Plot Group

Plot

Label: MOAT, comp1.maxdeform

Data

Dataset: Study 1/Solution 1 (sol1)

Title

Plot Settings

x-axis label: MOAT mean

y-axis label: MOAT standard deviation

Two y-axes

Flip the x- and y-axes

Axis

Manual axis limits

x minimum: 4.41014E-6

x maximum: 2.96564E-5

y minimum: 4.81035E-7

y maximum: 2.71982E-6

Preserve aspect ratio

x-axis log scale

y-axis log scale

Grid

Show grid

Manual spacing

x spacing: 1

y spacing: 1

Extra x:

Extra y:

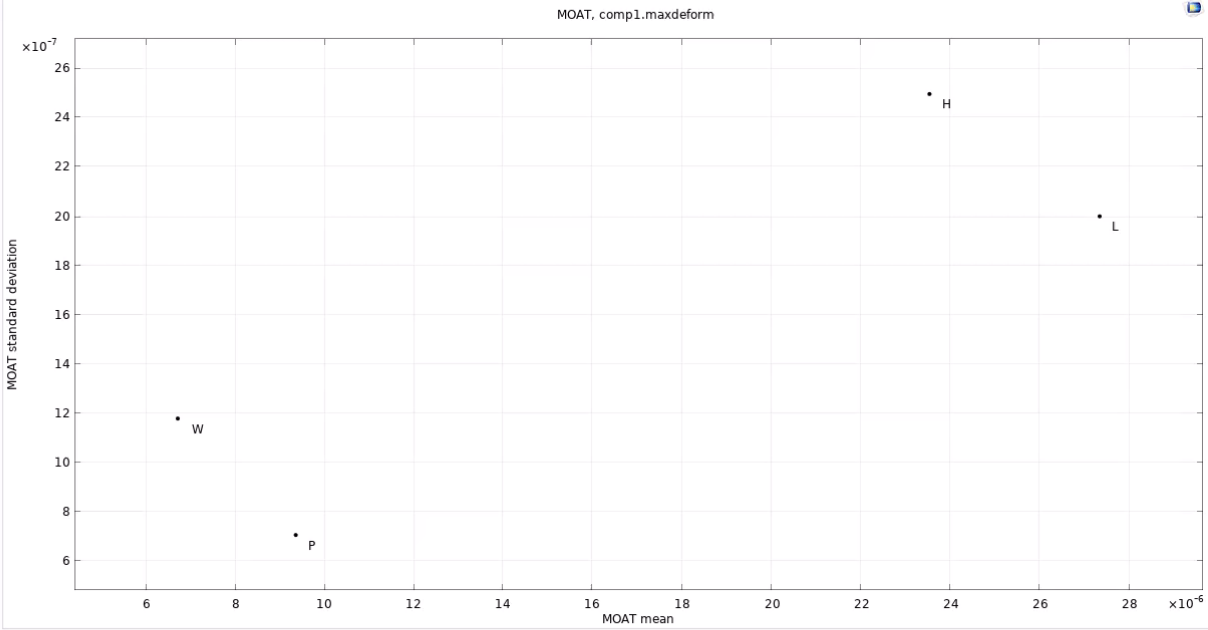
Legend

Show legends

Show maximum and minimum values

Position: Upper right

Graphics



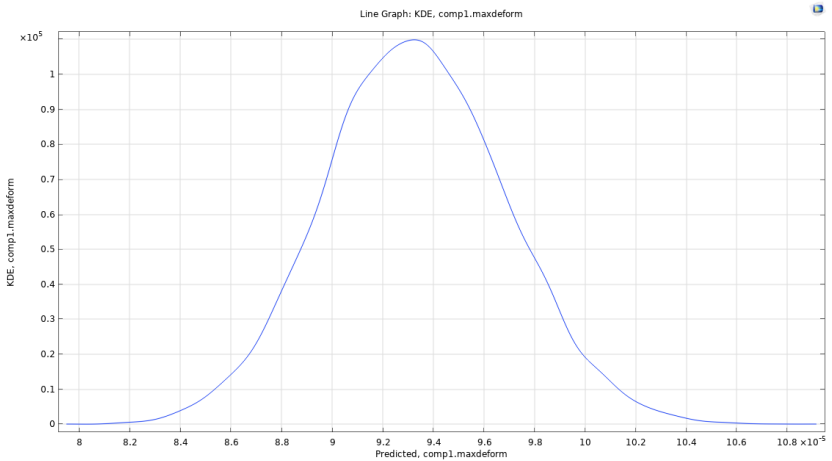
Messages Progress Log MOAT

[Jan 17, 2022, 8:33 PM] Number of degrees of freedom solved for: 16002.
[Jan 17, 2022, 8:33 PM] Number of degrees of freedom solved for: 17232.
[Jan 17, 2022, 8:33 PM] Number of degrees of freedom solved for: 16830.
[Jan 17, 2022, 8:33 PM] Number of degrees of freedom solved for: 16830.
[Jan 17, 2022, 8:33 PM] Solution time (Study 2): 32 s.
[Jan 17, 2022, 8:35 PM] Image copied to clipboard.

1.46 GB | 1.74 GB

Uncertainty Quantification Module Study Types

- Screening
- Sensitivity Analysis
- Uncertainty Propagation:
  - Higher computational cost
  - Determine output probability distribution given the input probability distributions
  - QoI confidence interval



	Mean	STD	Minimum	Maximum	Lower 90%	Upper 90%	Lower 95%	Upper 95%	Lower 99%	Upper 99%
comp1.maxdeform	9.3302E-5	3.5729E-6	7.9505E-5	1.0915E-4	8.7528E-5	9.9167E-5	8.6395E-5	1.0047E-4	8.4376E-5	1.0278E-4

The parameter input value may have a distribution where the property varies, for example, a Normal distribution. How will the output distribution look given a distribution in the input parameters?

Untitled.mph - COMSOL Multiphysics

File Home Definitions Geometry Materials Physics Mesh Study Results Developer Sobol index, comp1.maxdeform

Plot Plot In | Line Graph Point Graph Global | Table Graph Line Segments Histogram | Mesh Annotation | More Plots | Color Expression Filter Graph Marker | More Attributes | Image Animation

Plot | Add Plot | Attributes | Export

Model Builder

- Definitions
  - Variables 1
  - Maximum 1 (maxop1)
  - Boundary System 1 (sys1)
  - View 1
- Geometry 1
  - Work Plane 1 (wp1)
    - Plane Geometry
      - Rectangle 1 (r1)
      - Offset 1 (off1)
      - Difference 1 (dif1)
    - View 2
  - Extrude 1 (ext1)
  - Form Union (fin)
- Materials
  - Structural steel (mat1)
- Solid Mechanics (solid)
  - Linear Elastic Material 1
  - Free 1
  - Initial Values 1
  - Edge Load 1
  - Fixed Constraint 1
  - Mesh 1
- Study 1
- Study 2 - Screening MOAT
- Study 3 - Sensitivity analysis
  - Uncertainty Quantification
  - Study 1
  - Solver Configurations
  - Job Configurations
- Results
  - Uncertainty Quantification Graph 1
    - Sobol index, comp1.maxdeform
  - Uncertainty Quantification Graph
    - MOAT, comp1.maxdeform
  - Datasets
  - Derived Values
    - Global Evaluation 1
  - Tables
  - Stress (solid)
  - Applied Loads (solid)
    - Edge Loads (solid)
  - Export
  - Reports

Settings

1D Plot Group

Plot

Label: Sobol index, comp1.maxdeform

Data

Dataset: Study 1/Solution 1 (sol1)

Title

Plot Settings

x-axis label: Input parameter

y-axis label: First-order and total Sobol indices

Two y-axes

Flip the x- and y-axes

Axis

Manual axis limits

x minimum: 0.6368

x maximum: 4.3632

y minimum: -0.01083

y maximum: 0.53214

Preserve aspect ratio

x-axis log scale

y-axis log scale

Grid

Show grid

Manual spacing

x spacing: 1

y spacing: 1

Extra x:

Extra y:

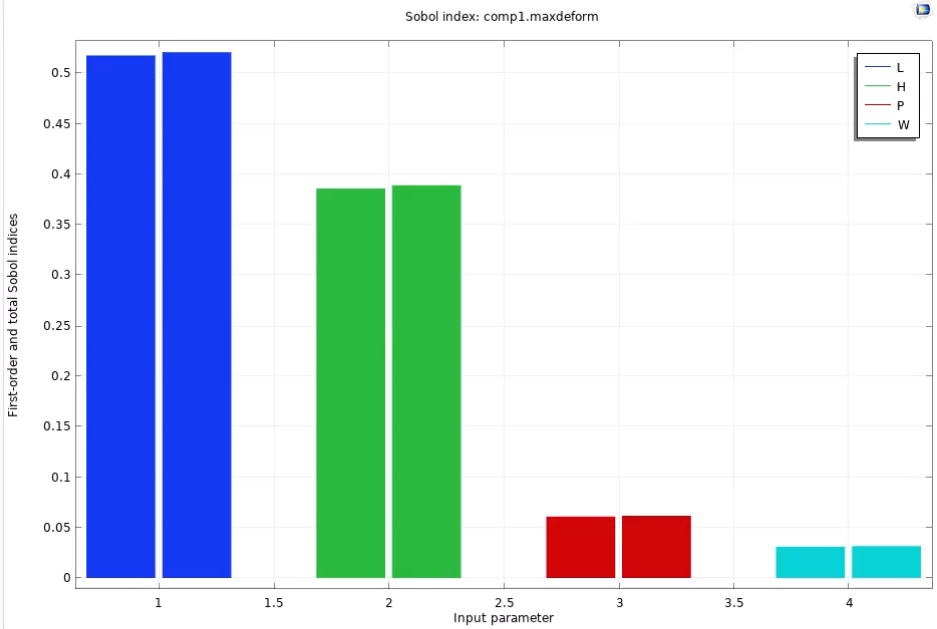
Legend

Show legends

Show maximum and minimum values

Position: Upper right

Graphics

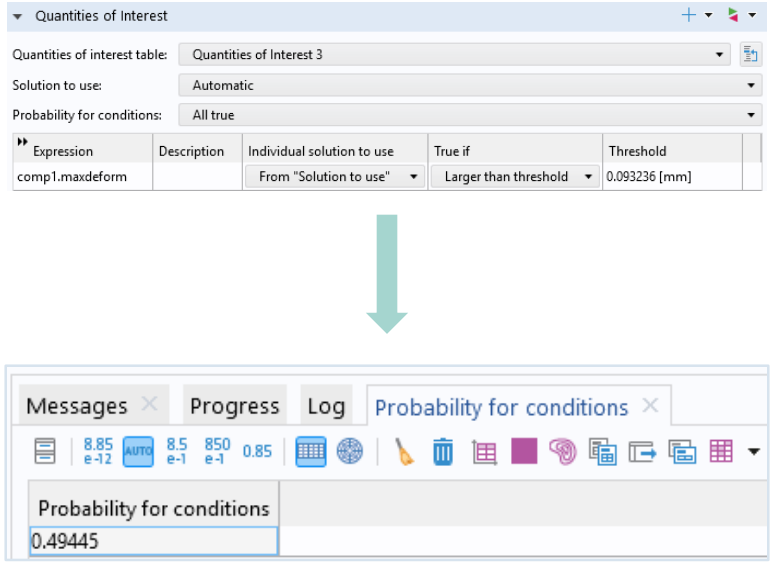


Messages | Progress | Log | Sobol

1.61 GB | 1.84 GB

Uncertainty Quantification Module Study Types

- Screening
- Sensitivity Analysis
- Uncertainty Propagation
- Reliability analysis:
  - Determine the probability that outputs satisfy reliability criteria
  - Reliability criteria e.g. maxdeform < 0.093236 mm



Given a nominal design and some specific uncertain inputs, what is the probability that the design fails? The failure can be a complete breakdown of the design, but it can also be phrased in terms of a quality criterion.

Untitled.mph - COMSOL Multiphysics

File Home Definitions Geometry Materials Physics Mesh Study Results Developer

Application Builder | Model Manager | Component 1 | Add Component | Parameters | Variables | Functions | Parameter Case | Import | LiveLink | Build All | Add Material | Solid Mechanics | Add Physics | Build Mesh | Mesh 1 | Compute | Study 4 - Uncertainty propagation | Add Study | Kernel density estimation, comp1.maxdeform | Add Plot Group | Windows | Reset Desktop

Workspace | Model | Definitions | Geometry | Materials | Physics | Mesh | Study | Results | Layout

Model Builder

- Untitled.mph *(root)*
  - Global Definitions
    - Parameters 1
    - Materials
  - Component 1 *(comp1)*
    - Definitions
    - Geometry 1
    - Materials
    - Solid Mechanics *(solid)*
    - Mesh 1
  - Study 1
  - Study 2 - Screening MOAT
  - Study 3 - Sensitivity analysis
  - Study 4 - Uncertainty propagation
    - Uncertainty Quantification
    - Study 1
    - Solver Configurations
    - Job Configurations
  - Results
    - Uncertainty Quantification Graph 2
      - Kernel density estimation, comp1.r
    - Uncertainty Quantification Graph 1
    - Uncertainty Quantification Graph
    - Datasets
    - Derived Values
    - Tables
      - Uncertainty propagation
        - UP predicted QoI
        - QoI confidence interval
        - Adaptive UP predicted QoI
        - Adaptive QoI confidence interv
        - UP predicted STD
        - Maximum entropy
        - Adaptive UP predicted STD
        - Adaptive maximum entropy
      - Sensitivity Analysis Sobol
      - Screening, MOAT
      - Table 1
      - Quantities of Interest
      - Quantities of Interest 1
      - Quantities of Interest 2
    - Stress (solid)
    - Applied Loads (solid)
    - Export

Settings

Table

Update

Label: QoI confidence interval

Data

Presentation format: All

Import...

Storage

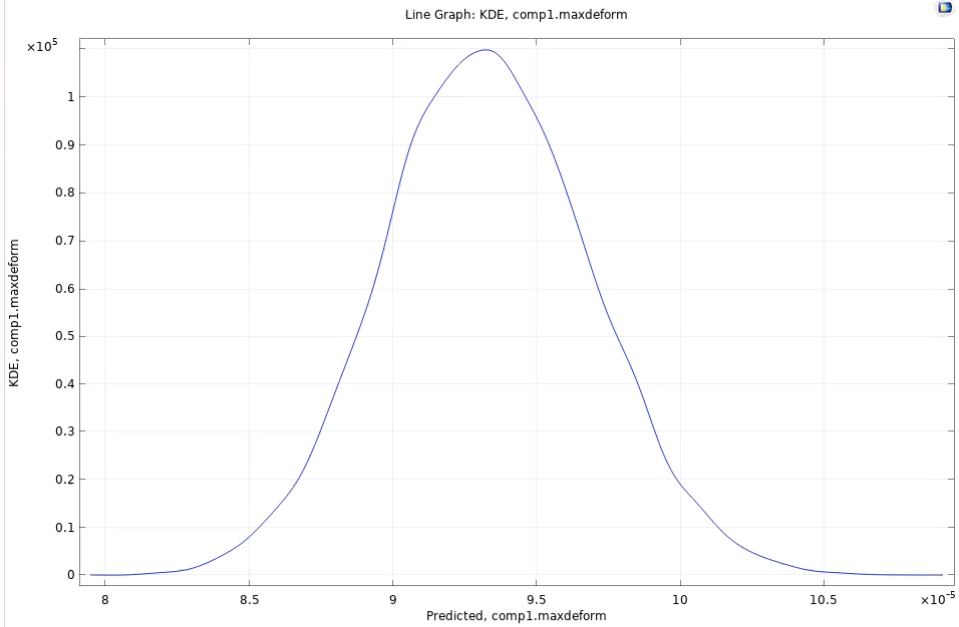
Store table: In model

Maximum number of rows: 10000

Column Headers

Column	Header
1	Mean
2	STD
3	Minimum
4	Maximum
5	Lower 90%
6	Upper 90%
7	Lower 95%
8	Upper 95%
9	Lower 99%

Graphics



Messages | Progress | Log | QoI confidence interval

	Mean	STD	Minimum	Maximum	Lower 90%	Upper 90%	Lower 95%	Upper 95%	Lower 99%	Upper 99%
comp1.maxdeform	9.3302E-5	3.5729E-6	7.9505E-5	1.0915E-4	8.7528E-5	9.9167E-5	8.6395E-5	1.0047E-4	8.4376E-5	1.0278E-4

2.02 GB | 2.37 GB

Thank you for your attention

You can find a video screen record of my model settings. Visit our **Youtube** channel and find „*Představení a ukázka novinek v COMSOL Multiphysics 6.0*“

https://youtu.be/y2yWmfn4leQ